

# Mathematics for New Technologies in Finance

## Exercise sheet 1

### Exercise 1.1

- (a) What is the formal definition of shallow or deep neural networks? (You might need the words affine function and activation function in the definition)
- (b) Why we always like to consider non-linear activation functions?
- (c) If the activation functions of neural network are bounded, e.g.  $x \mapsto \tanh(x)$ , is the neural network bounded for all possible inputs? If the activation functions are unbounded, e.g. ReLU, is the neural network bounded on compact input spaces?
- (d) Is the sum of two neural networks still a neural network? Is the product of two neural networks still a neural network? Is the composition of two neural networks still a neural network?
- (e) Are neural networks with ReLU activation functions differentiable?

### Exercise 1.2

- (a) Build a tent neural network  $h: \mathbb{R} \rightarrow \mathbb{R}$  s.t.

$$h(x) = \begin{cases} x + 1, & x \in [-1, 0] \\ 1 - x, & x \in [0, 1] \\ 0, & \text{otherwise} \end{cases}, \quad (1)$$

from a ReLU neural network.

- (b) Use (a) to prove the universal approximation theory on  $[0, 1]$  i.e. every continuous function  $f$  on  $[0, 1]$  can be uniformly approximated by neural networks (see the Faber-Schauder expansion of a continuous function).

### Exercise 1.3 See the notebook

- (a) Code a neural network to approximate a function on  $[-5, 5]$ .
- (b) Code a tent neural network on  $\mathbb{R}^2$ .

**Exercise 1.4** Prove the real continuous function on compact set with supremum norm i.e.  $(C_0(K), \|\cdot\|_\infty)$  is a Banach space.