D-ARCH Mathematics HS 2020
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Midterm exam

1. **Area enclosed by two curves**

   Let $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ with

   $$f(x) = x^3 - x\,, \quad \text{and} \quad g(x) = x^2 - 1\,.$$

   (a) Determine the points $x_1 < x_2$ in which the graphs of f and g intersect.

   (b) Compute the area that is enclosed by the graphs of f and g between x_1 and x_2.

2. **Complex numbers**

   Using complex numbers, verify that for any $x \in \mathbb{R}$

   $$\cos(3x) = \cos(x)\big(4\cos^2(x) - 3\big)\,.$$

3. **First order differential equation**

   Find the constant C and the solution $y(x)$ of the differential equation

   $$y' + xy + Cx = 0$$

   such that $y(0) = 0$ and $y\left(\sqrt{2}\right) = \frac{1}{e} - 1$.

4. **Linear differential equation with constant coefficients**

   Find the solution $y(x)$ of the differential equation

   $$y'' - 4y' + 4y = \sin(x)$$

   that satisfies the initial conditions $y(0) = \frac{1}{5}$ and $y'(0) = 1$.