

1. Area enclosed by two curves

Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(x) = x^3 - x^2 - 4x + 2, \quad \text{and} \quad g(x) = x^2 - 3x.$$

- (a) Determine the points $x_1 < x_2 < x_3 \in \mathbb{R}$ in which the graphs of f and g intersect.
- (b) Compute the area that is enclosed by the graphs of f and g between x_1 and x_3 .

Hint: It may help to multiply $(x + 1)(x - 1)(x - 2) =: h(x)$.

2. Extrema

Consider the function

$$f : [-1, 1] \rightarrow \mathbb{R} : \quad f(x) = x^2 \left(\frac{1}{4}x^2 + \frac{1}{3}x - 1 \right).$$

For which $x \in [-1, 1]$ is $f(x)$ minimized and for which x is it maximized?

3. Complex numbers

Find all the solutions of the equation

$$z^5 = 4(1 + i).$$

4. First order differential equation

Find the solution $x(t)$ of the differential equation

$$x' = 2xte^t$$

subject to the condition that $x = 1$ when $t = 0$.