INTEGRATION BY SUBSTITUTION & BY PARTIAL FRACTION DECOMPOSITION, THE FUNDAMENTAL THEOREM OF CALCULUS

1. Compute the following integrals by using the method of substitution. Notice that some of these integrals were already given in the last problem set.

   (a) $\int 2x\sin(x^2+1)\,\mathrm{d}x$ (b) $\int e^{-10x}\,\mathrm{d}x$ (c) $\int_0^{\pi/5} \cos(5x-\frac{\pi}{2})\,\mathrm{d}x$

   (d) $\int \frac{e^{-\sqrt{x+1}}}{\sqrt{x+1}}\,\mathrm{d}x$ (e) $\int_2^7 \frac{1}{x+3}\,\mathrm{d}x$ (f) $\int_3^5 \frac{1}{2x-2}\mathrm{d}x$

2. Compute the following integrals by using the method of partial fraction decomposition.

$$\text{(a)} \int_2^3 \frac{x-1}{x\,(x^2-2)},\mathrm{d}x, \qquad \text{(b)} \int_{-1}^1 \frac{x^2}{(x+2)(x+3)^2}\mathrm{d}x,$$

3. Compute the following integrals by choosing the right method. Check your solutions by differentiating the results.

$$\text{(a)} \int x\cos x\sin x\,\mathrm{d}x, \qquad \text{(b)} \int \frac{1}{x^2(x^2-1)}\,\mathrm{d}x, \qquad \text{(c)} \int \frac{1}{x\ln x}.$$

4. The following exercise gives some further practice in both differentiation and integration. Notice how the variable t vanishes in (a)-(c), such that the final function depends only on x.

   (a) Compute the integral $f_1(x) = \int_0^x \frac{e^{-\sqrt{t}}}{\sqrt{t}}\,\mathrm{d}t$ and evaluate $f_1'(1)$.

   (b) Compute the integral $f_2(x) = \int_x^{x^2} \frac{e^{-\sqrt{t}}}{\sqrt{t}}\,\mathrm{d}t$ and evaluate $f_2'(1)$.

   (c) (optional) Prove using the chain rule and the first fundamental theorem of calculus $\frac{d}{dx}(\int_0^x f(t)dt) = f(x)$ that

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{g(x)}^{h(x)} f(t)\,\mathrm{d}t = f(h(x))\cdot h'(x) - f(g(x))\cdot g'(x)$$

   (d) Compute $f_1'(1)$ and $f_2'(1)$ without integration.

   (e) Compute $g_1'(2)$ for

$$g_1(x) = \int_0^x \frac{dt}{t^3+1}.$$

   (f) Compute $g_2'(2)$ for

$$g_2(x) = \int_0^{x^2} \frac{dt}{t^3+1}.$$

5. An extended period of rain in an area causes a waterflow. After t days, the flow is $f(t)$ mm of water per day, modelled by the function

$$f(t) = 25e^{-t} + e^{-0.05t}.$$

   How much water flows from the area in the first ten days?