DIFFERENTIAL CALCULUS

1. Compute the first derivative of

$$\text{(a) } x^8 e^{-x^3} - x - 100, \qquad \text{(b) } \frac{\ln(\sin^2(x))}{\cos(x)}, \qquad \text{(c) } \arctan(\sqrt{x}).$$

2. Compute the first derivative of the two functions

   (a) $f(x) = e^{\sin(x^3+\cos(x^2))}$,

   (b) $g(x) = \cos^2\left(\frac{x^3+1}{x^2+1}\right)$.

3. For which $x \in \mathbb{R}$ does the graph of $f : \mathbb{R} \to \mathbb{R}$ with

$$y = f(x) = e^{\sin x} \cdot e^{\cos x}$$

   have horizontal tangents? These are tangents of the form $t(x) = a$ with $a \in \mathbb{R}$.

4. Compute the second derivative $h''(x) = (h'(x))'$ of the function

$$h(x) = \ln(\ln(x)).$$

5. Suppose that a function f is continuous and differentiable in the interval $[0, 1]$. Suppose further that $f(0) = -1$ and $f'(x) \leq 2$ for all $x \in [0, 1]$. What is the largest possible value of $f(1)$?

6. Let f be a differentiable function. Compute the expression

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x^3)}{xf(x^2)}\right).$$