DIFFERENTIAL CALCULUS

All questions using MATLAB are optional. We encourage you to familiarize yourself with MATLAB, as you might ahve to use it later in other courses. For installation, you can follow his tutorial https://www2.math.ethz.ch/education/bachelor/lectures/fs2013/other/num_meth_itet/Matlab.html

1. Plot the two functions $f(x) = 4x^3 + 2x^2 - 5x - 2$ and $g(x) = 2x^2 - x - 2$ for $x \in [-2, 2]$ in MATLAB, using the command `fplot`.
   For help, see https://ch.mathworks.com/help/matlab/ref/fplot.html.

   (a) Determine the three x-coordinates $x_1 < x_2 < x_3$ of the points, where the graphs of the two functions $f(x)$ and $g(x)$ intersect.

   (b) On the x-axis of your plot, let $x_{\min}$ and $x_{\max}$ correspond to the local minimum and the local maximum of f. Compute by hand $x_{\min}$ and $x_{\max}$.

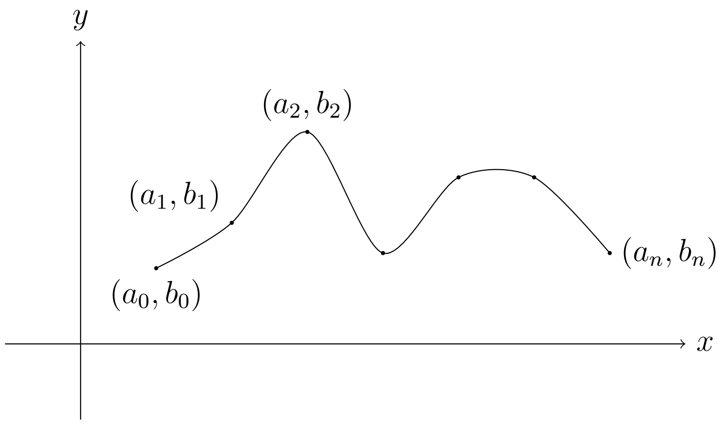
2. Find the global minimum and the global maximum of the function
$$f(x) = x^4 - 4x^3 + 4x^2 - 3$$
   on the interval $[-2, 3]$. Sketch the function by hand and using MATLAB.

3. Find the equation of the line that is perpendicular to the tangent to the curve
$$y(x) = (x^4 - 1)^3 \ln(x + 1)$$
   at the origin. Sketching by hand the curve first will help.

4. Determine the line that is tangent to the curve
$$f(x) = \frac{e^x \sin(x)}{\cos(x)} + 1$$
   at the point $x = 0$.

5. Let $f(x)$ be a continuous and differentiable function on the interval $[-7, 0]$ that satisfies $f(-7) = -3$. Compute the largest possible value for $f(0)$ if $f'(x) \leq 2$.

6. You are in charge of designing a roller coaster ride. This simple ride is completely described, as viewed from the side, by the following figure.



You are given the set of points $(a_0, b_0), (a_1, b_1), \ldots, (a_n, b_n)$ and you need to connect these in a reasonably smooth way. A method often used is to find polynomials $f_k(t)$ of degree at most 3, which define the shape of the ride between (a_{k-1}, b_{k-1}) and (a_k, b_k) by requiring $f_k(a_{k-1}) = b_{k-1}$ and $f_k(a_k) = b_k$ as well as $f'_k(a_k) = f'_{k+1}(a_k)$, $f''_k(a_k) = f''_{k+1}(a_k)$. Explain the practical significance of these conditions. For the convenience of the riders, it is also required that $f'_1(a_0) = f'_n(a_n) = 0$. Why?

Show that satisfying all these conditions amounts to solving a linear system (without solving the system)!

Note: The aim of this exercise is to help you visualise and understand the concept of a derivative. You don't have to write anything down (but you can if it helps!), just think about it.