DIFFERENTIAL CALCULUS

1. Compute the following integrals by finding an appropriate antiderivative.

   (a) $\displaystyle\int_{-2}^{2} (x^3 + 8x)\,\mathrm{d}x$ (b) $\displaystyle\int e^{-7x}\,\mathrm{d}x$ (c) $\displaystyle\int \sqrt{5x}\,\mathrm{d}x$

   (d) $\displaystyle\int_{0}^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x$ (e) $\displaystyle\int_{2}^{8} \frac{1}{x}\,\mathrm{d}x$ (f) $\displaystyle\int \mathrm{d}x$

2. Compute the following integrals using integration by parts.

   (a) $\displaystyle\int \cos x \ln(\sin x)\,\mathrm{d}x$ (b) $\displaystyle\int \frac{x}{\cos^2 x}\,\mathrm{d}x$ (c) $\displaystyle\int x^3 e^x\,\mathrm{d}x$

   (d) $\displaystyle\int \ln(x^2+1)\,\mathrm{d}x$ (e) $\displaystyle\int x \ln x\,\mathrm{d}x$ (f) $\displaystyle\int \sin^2 x\,\mathrm{d}x.$

3. Consider as last week the two functions

   $$f(x) = 4x^3 + 2x^2 - 5x - 2 \qquad \text{and} \qquad g(x) = 2x^2 - x - 2.$$

   (a) Determine the x-coordinates $x_1 < x_2 < x_3$ of the points where the graphs intersect.

   (b) Calculate the integral $\displaystyle\int_{x_1}^{x_3} \big(f(x) - g(x)\big)\,\mathrm{d}x$.

   (c) Calculate the area of the shaded region.

4. Calculate the area of the region bounded by the line $x = 5$, the curve $y = \sqrt{x}$ and the x-axis.

5. For which $x \in (0, \frac{3\pi}{2})$ is $f(x) = \int_x^{2x} \frac{\sin t}{t} dt$ a local maximum?