Mathematics

Problem Set 6

DIFFERENTIAL MULTIVARIABLE CALCULUS

1. For the following functions, first determine the most efficient way of computing $f_{xy} = f_{yx}$ and then differentiate accordingly: should you differentiate with respect to x or to y first?

   (a) $f(x, y) = x\sin(y) + e^y$,

   (b) $f(x, y) = \frac{1}{x}$,

   (c) $f(x, y) = y + \frac{x}{y}$,

   (d) $f(x, y) = y + x^2y + 4y^3 - \ln(y^2 + 1)$,

   (e) $f(x, y) = x^2 + 5xy + \sin(x) + 7e^x$,

   (f) $f(x, y) = x\ln(xy)$.

2. For each of the following functions, sketch a typical level curve.

   (a) $f(x, y) = y^2$, (b) $f(x, y) = 1 - |x| - |y|$, (c) $f(x, y) = \sqrt{x^2 + y^2 - 9}$.

3. For each of the following functions, compute the gradient $\nabla f(x, y)$.

   (a) $f(x, y) = \sqrt{x^2 + y^2 - 9}$, (b) $f(x, y) = xy$, (c) $f(x, y) = x^3 + 3(x^2 - y^2) - 3$.

4. Find the line that is tangent to the intersection of $z = \arctan(xy)$ with the plane $x = 2$ at $(2, 1/2, \pi/4)$.

5. The lengths a, b, c of the edges of a rectangular box are changing with time. At some fixed time t_0 we have $a = 1$, $b = 2$, $c = 3$, and $\frac{da}{dt} = \frac{db}{dt} = 1$, $\frac{dc}{dt} = -3$.

   At what rates are the volume and the surface area of the box changing at t_0? Are the interior diagonals of the box increasing or decreasing in length?

6. Let x, y be non-negative real numbers such that $x+y = 12$. What is the minimum possible value of x^2y ? For which values of x and y is this minimum attained?

7. Find the tangent plane and normal line to $x^2+y^2+z^2 = 30$ at the point $(1, -2, 5)$.