PARTIAL DERIVATIVES AND DIFFERENTIAL EQUATIONS OF FIRST ORDER

1. Calculate dz/dt for each of the following functions:
   (a) $z = f(x, y) = 4x^2 + 3y^2$, where $x = x(t) = \sin(t)$, $y = y(t) = \cos(t)$,
   (b) $z = f(x, y) = \sqrt{x^2 - y^2}, x = x(t) = e^{2t}, y = y(t) = e^{-t}$.

2. Calculate dz/du and dz/dv using the following functions:

$$z = f(x, y) = 3x^2 - 2xy + y^2,$$

   where $x = x(u, v) = 3u + 2v,\ y = y(u, v) = 4u - v$.

3. Determine $D_{\vec{u}} f$ for $f(x, y) = \cos(xy)$ in the direction of $\vec{v} = (3, -4)$ where we recall that $\vec{u} = \vec{v}/|\vec{v}|$.

4. For each of the following differential equations, find the general solution and sketch the integral curves.
   (a)
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1 + x^2}},$$
   (b)
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x}{(1 + x^2)^{1/3}}.$$

5. (a) Find all solutions of the differential equation $y\,dx = x\,dy$, and draw the integral curves in the plane.
   (b) Describe geometrically (in words) the set of curves that are orthogonal to the integral curves.
   (c) Describe these orthogonal curves algebraically, by providing appropriate equations. Check that they satisfy $\frac{\mathrm{d}x}{\mathrm{d}y} = -\frac{y}{x}$. Can you explain why this is to be expected?

6. A patient initially has 6 million bacteria in his system, before starting a penicillin treatment. After x days, the rate of change is proportional to the total number of million bacteria and $0.1 \cdot (4 - 2x)$.
   (a) Find the total number of bacteria $y(x)$ after x days.
   (b) Sketch the graph of $y(x)$ and explain the curve.
   (c) Find the highest value of $y(x)$ and when it occurs.