

ORDINARY DIFFERENTIAL EQUATIONS

1. Let α be a real number and consider the initial value problem (IVP)

$$\frac{dy}{dx} = y^\alpha, \quad y(0) = 0.$$

- (a) Show that this IVP has no solution if $\alpha = 1$.
- (b) Determine the integral curve when $\alpha \neq 1$.
- (c) Find the condition on α for which the system has a solution $y(x)$ defined for all $x \geq 0$. Can you find an α for which two solutions exist?

2. Solve the following second order differential equations:

- (a) $2y'' + 7y' = 4y$,
- (b) $y'' + 2y' + y = 0$,
- (c) $y''' - y'' - 9y' + 9y = 0$.

3. For which values of the parameter $\alpha \in \mathbb{R}$ does the differential equation

$$y'' + y' - 6y = -2\alpha e^x$$

have solutions that are bounded for $x \rightarrow \infty$? Determine all these solutions.

4. Use complex variables to simultaneously find the general solutions of the two equations

$$y''_a + 2y'_a + 2y_a = \cos t, \quad y''_b + 2y'_b + 2y_b = \sin t.$$

5. Determine the general solutions of the following differential equations:

- (a) $y'' + 3y' + 2y = 2e^{-t}$,
- (b) $y'' + 2y' + 2y = 5 \cosh t$,
- (c) $y'' + 3y' + 2y = 2 \cosh t$.

6. Determine the solution of the differential equation

$$y' + \frac{2x}{x^2 + 1} \cdot y = 4x$$

that passes through the point $(x, y) = (1, 3)$.

7. Solve the following first order differential equations.

(a) $y' - y = \cos x$ subject to the initial condition $y(0) = 0$.

(b) $y' = 5x - \frac{3y}{x}$ with initial condition $y(1) = 2$.

(c) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, with no specified initial condition.