Mathematics

IBS

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Introduction: an example

Natasha and Raúl are on a bike tour around a lake. They both have constant velocity.
One turn around the lake has a length of 14 km.
If they both go into the same direction they meet every 105 minutes.
If they ride in opposite direction, they meet every 15 minutes.
What are their velocities?

We assume that, at the time $t = 0$, Natasha and Raúl start at the same point.
Let v_i, $i = 1, 2$, denote the velocities and $r_i(t) = v_i \cdot t$, $i = 1, 2$, the distance.

First equation

If they go in opposite direction, they meet when the sum of their distances r_i, $i = 1, 2$, to the starting point equals the total length 14 km.

Hence, after $t_1 = 15$ min,

$$r_1(t_1) + r_2(t_1) = 14 \, km$$

and, with $r_i(t) = v_i \cdot t$, we get

$$v_1 \cdot \frac{1}{4} \, h + v_2 \cdot \frac{1}{4} \, h = 14 \, km$$

since 15 min $= \frac{1}{4}$ h.

Second equation

Now they ride in the same direction. They meet again if their distances differ by 14 *km*, hence, after $t_2 = 105$ min,

$$r_1(t_2) - r_2(t_2) = 14\,km$$

and, with $r_i(t) = v_i \cdot t$, we get

$$v_1 \cdot \frac{7}{4}\,h - v_2 \cdot \frac{7}{4}\,h = 14\,km$$

since 105 min $= \frac{7}{4}$ h.

The system

We solve

$$v_1 \cdot \frac{1}{4} + v_2 \cdot \frac{1}{4} = 14$$

$$v_1 \cdot \frac{7}{4} - v_2 \cdot \frac{7}{4} = 14$$

This is equivalent to

$$\frac{1}{4}(v_1 + v_2) = 14$$

$$\frac{7}{4}(v_1 - v_2) = 14$$

Solve the system

Now the system

$$\frac{1}{4}(v_1 + v_2) = 14$$

$$\frac{7}{4}(v_1 - v_2) = 14$$

is equivalent to

$$v_1 + v_2 = 56$$
$$7v_1 - 7v_2 = 56$$

and

$$v_1 + v_2 = 56$$
$$v_1 - v_2 = 8$$

Solve the system

$$v_1 + v_2 = 56$$
$$v_1 - v_2 = 8$$

Adding both equations we get $2v_1 = 64$ and subtraction yields $2v_2 = 48$. Hence the velocities of Natasha and Raúl are

$$v_1 = 32 \, km/h \quad \text{and} \quad v_2 = 24 \, km/h.$$