

Trigonometric functions

For any $\alpha \in \mathbb{R}$:

$$e^{i\alpha} = \cos \alpha + i \sin \alpha$$

Periodicity

For $k \in \mathbb{Z}$ we have

$$\sin(\alpha + k 2\pi) = \sin \alpha$$

$$\cos(\alpha + k 2\pi) = \cos \alpha$$

$$\tan(\alpha + k 2\pi) = \tan \alpha$$

$$\cot(\alpha + k 2\pi) = \cot \alpha$$

Evaluate

$\sin(-\alpha) = -\sin \alpha$	$\cos(-\alpha) = \cos \alpha$	$\tan(-\alpha) = -\tan \alpha$
$\sin(\pi - \alpha) = \sin \alpha$	$\cos(\pi - \alpha) = -\cos \alpha$	$\tan(\pi - \alpha) = -\tan \alpha$
$\sin(\pi + \alpha) = -\sin \alpha$	$\cos(\pi + \alpha) = -\cos \alpha$	$\tan(\pi + \alpha) = \tan \alpha$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	
$\cot x$		$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

Relations between functions of same angle

$\cos^2 \alpha + \sin^2 \alpha = 1$	$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$	$\frac{1}{\cos^2 \alpha} = 1 + \tan^2 \alpha$
	$\cot \alpha = \frac{1}{\tan \alpha}$	$\frac{1}{\sin^2 \alpha} = 1 + \cot^2 \alpha$

Addition, subtraction and multiples of angles

$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$	$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$
$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$	$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$	$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

$\sin 2\alpha = 2 \sin \alpha \cos \alpha$	$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1$
$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$	

$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$	$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$	$\tan^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$
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$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$

Sum of trigonometric functions

$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$	$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$
$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$	$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2}$
$\tan \alpha + \tan \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cdot \cos \beta}$	$\tan \alpha - \tan \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cdot \cos \beta}$