Aufgabe 1. Show by induction the following identities for integers $n \geq 1$:

a) $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$,

b) $1^3 + 2^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2$.

Aufgabe 2. Draw V points on a sheet of paper. Connect the points with enough lines (which do not intersect and have different start and end points) so that you get a connected picture. That is, there is a path from each point to every other point along the drawn lines. Let E be the number of lines and F the number of areas into which the lines divide your sheet of paper. Calculate,

$$V - E + F$$

for different examples and make a conjecture. Prove the conjecture by induction.

Aufgabe 3. Find a relation on the natural numbers $\mathbb{N}$ fulfilling the following properties:

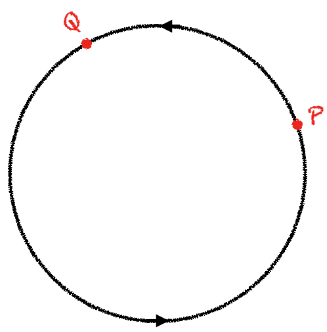
a) only the symmetry,

b) the transitivity (and the antisymmetry),

c) the reflexivity and the transitivity but not the symmetry .

Aufgabe 4. Let $(X, \leq)$ be an ordered set. If there exists $m \in X$ such that $x \leq m$ holds for all $x \in X$, then $m \in X$ is called a/the **maximum** of X. Convince yourself that X can have at most one maximum.

Aufgabe 5. We consider the set of all points on a circular line in the plane $\mathbb{R}^2$. For two points P and Q we say that $P \leq Q$ holds if $P = Q$, or if the arc from P to Q in the counterclockwise direction is (strictly) shorter than the arc from Q to P in the counterclockwise direction.



Is the relation defined in this way an order relation on the points of the circular line? Can there be an order relation on this set at all?