Exercise 1. (Important)

a) Watch some videos on the Youtube channel Polyquity EPFL.

b) Go to the website of Respect @ ETH Zurich and find out what you can do if you experience or observe something similar.

Exercise 2. Let $f : [1, 2] \to \mathbb{R}$ be the function given by $f(x) = x$.

a) Prove *without using the fundamental theorem* that

$$\int_1^2 f(x)dx = \frac{3}{2}.$$

b) Now use the fundamental theorem to show that

$$\int_1^2 f(x)dx = \frac{3}{2}.$$

Note: This exercise is intended to show how useful the fundamental theorem is when calculating integrals. The calculation of integrals with the fundamental theorem will then be the topic of exercise sheet 11.

Exercise 3. Let $f : [a, b] \to \mathbb{R}$ be a continuous function. Prove that

$$f = 0 \iff \int_a^b |f(x)|dx = 0.$$

Exercise 4. Let $f, g : [a, b] \to \mathbb{R}$ be continuous functions such that

$$\int_a^b f(x)dx = \int_a^b g(x)dx.$$

Show that there is a $y \in [a, b]$ such that $f(y) = g(y)$.

Hint: Use Exercise 3.

Exercise 5. (Easy) Let $a < b$ and $f : [a, b] \to \mathbb{R}$ be a continuous function. Show that there exists a $y \in [a, b]$ such that

$$f(y) = \frac{1}{b-a} \int_a^b f(x)dx.$$

Hint: Use Exercise 4.

Exercise 6. (Easy) Let $f : [a,b] \to \mathbb{R}$ be integrable, and let $f^* : [a,b] \to \mathbb{R}$ be a function obtained by changing the value of f at finitely many points in $[a,b]$. Show that f^* is Riemann-integrable and that

$$\int_a^b f\,dx = \int_a^b f^*\,dx.$$

Exercise 7. (Hard) Let $f : [0,1] \to \mathbb{R}$ be an integrable function and $\varepsilon > 0$. Show that there exists a continuous function $g : [0,1] \to \mathbb{R}$ such that

$$\int_0^1 |f(x) - g(x)| dx < \varepsilon.$$

Note: The aim of this Exercise is to show that every Riemann-integrable function can be approximated "arbitrarily well" with a continuous function.