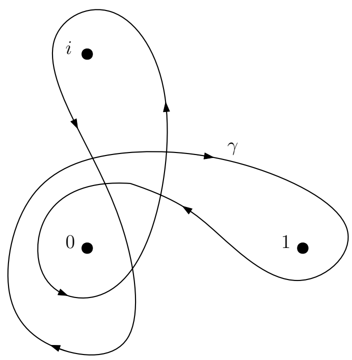
Exercises with a $\star$ are eligible for bonus points.

11.1. $\star$ **Complex integral** Evaluate

$$\int_{|z|=1} \frac{z^{10} - 2iz}{2\pi z^{11} + 2z^6 - 3z^4 - i}\, dz.$$

Hint: take advantage of Rouché Theorem and the Homotopy Theorem.

11.2. Winding number Evaluate the integral $\int_\gamma f\, dz$ when $f(z) = \frac{e^{iz}}{z^2(z^4-1)}$ and γ is as follows:



11.3. Fractional Residues Prove the following: if z_0 is a simple pole of a meromorphic function f and A_ε is an arc of the circle $\{z \in \mathbb{C} : |z - z_0| = \varepsilon\}$ of an angle $\alpha \in (0, 2\pi]$, then

$$\lim_{\varepsilon \to 0} \int_{A_\varepsilon} f\, dz = \alpha i \operatorname{res}_{z_0}(f).$$

11.4. $\star$ **Real integral** Evaluate

$$\int_{-\infty}^{+\infty} \frac{\sin(x)}{x(x-\pi)}\, dx.$$

Hint: take a suitable contour in $\mathbb{C}$ that avoids the zeros of the denominator. Take advantage of Exercise 11.3.

11.5. Real integral II Let $\alpha \in (0, 1)$. Evaluate

$$\int_{0}^{+\infty} \frac{x^{2\alpha - 1}}{1 + x^2}\, dx,$$

choosing a suitable branch of the logarithm.