Homology, extra exercises

1. Recall that the augmentation map $\epsilon : C_0(X) \to \mathbb{Z}$ takes a 0-chain $\sum_i n_i \sigma_i$ to the integer $\sum_i n_i$. Prove that if X is non-empty and path connected then ϵ induces an isomorphism $\mathrm{H}_0(X) \to \mathbb{Z}$.

2. Find a way of identifying pairs of faces of Δ^3 (standard 3-simplex) to produce a Δ-complex structure on S^3 having a single 3-simplex, and compute the simplicial homology groups of this Δ-complex.

3. Compute the homology groups of the Δ-complex X obtained from Δ_n by identifying all faces of the same dimension. Thus X has a single k simplex for each $k \leq n$.

4. Show that if A is a retract of X then the map $\mathrm{H}_n(A) \to \mathrm{H}_n(X)$ induced by the inclusion $A \subset X$ is injective.

5. Show that chain homotopy of chain maps is an equivalence relation.