CW-COMPLEXES & CELLULAR HOMOLOGY

closure finite

weak topology

these are topological spaces K built inductively.

We start with $K^{(0)}$ – a discrete set of points, called 0-cells. Suppose we defined $K^{(i)}$, $0 \le i \le n-1$. Let I_n be some index set. $\forall \sigma \in I_n$ take a copy of B^n (closed n-dim ball), which we denote by B^n_σ.

$$\partial B^n_\sigma := S^{n-1}_\sigma .$$

$$Y := \bigsqcup_{\sigma \in I_n} B^n_\sigma , \quad \partial Y = \bigsqcup_{\sigma \in I_n} S^{n-1}_\sigma .$$

$\forall \sigma \in I_n$, let $f_{\partial\sigma} : S^{n-1}_\sigma \to K^{(n-1)}$ be a map.

notation

Define $K^{(n)} := \left(K^{(n-1)} \sqcup \coprod \right) / \sim$

$\forall \sigma \in I_n, y \in S^{n-1}_\sigma$

$y \sim f_{\partial\sigma}(y)$.

$f_{\partial\sigma}$ is called the **ATTACHING MAP** for the cell σ. B^n_σ is also called the **CELL** corresponding to σ.

The process could stop or be infinite

in this case

$K = K^{(n_0)}$ for some n_0

If it is infinite, $K^{(0)} \subset K^{(1)} \subset \ldots \subset K^{(n)} \subset \ldots$

Put $K = \bigcup_{n \geq 0} K^{(n)}$.

$K^{(n)} \subset K$ is called the **n-skeleton** of K.

If $\nexists$ cells of dimension higher than n_0, we say K is a **FINITE DIMENSIONAL** CW-complex.

$\forall \sigma \in I_n$, denote by $f_\sigma : B^n_\sigma \to Y \to K^{(n)} \subset K$ the map that extends $f_{\partial\sigma}$ & is a homeomorphism from the interior of $\mathring{B}^n_\sigma$ onto its image.

f_σ is called the characteristic map of the cell σ. Put

$$K_\sigma := f_\sigma(B^n_\sigma)$$

closed cell

Note: K_σ might not be homeomorphic to B^n.

Put $U_\sigma := f_\sigma(B^n_\sigma \setminus S^{n-1}_\sigma) \subset K_\sigma$

open cell

$$U_\sigma \underset{\text{via } f_\sigma}{\approx} \operatorname{Int} B^n \approx \mathbb{R}^n.$$