SOME GUIDELINES FOR PREPARING FOR A BSC/MSC THESIS OR SEMESTERARBEIT

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Abstract. A selection of advice and guideline for the writing of bachelor or master-level theses.

(1) Usually, you start by getting from an advisor or mentor a topic for your thesis, and a first list of references for the material involved. You should immediately look for the "best" survey or textbook-level presentations of the material (either of the precise topic, or of the general area involved).

Your advisor is often the best person to suggest such references, but it also often pays to proactively search the internet or the library.

(2) You can rather quickly start a "skeleton" LaTeX file with some basic structure (introduction, statements, proofs, bibliography, etc). This can be filled as you learn and build your understanding of the subject, but you must also be prepared to re-organize and change the presentation as your knowledge expands. It is convenient to organize such a file with a main LaTeX file which inputs a series of shorter files specific to each part of the manuscript.

(3) Part of the thesis topic might involve learning about "new" mathematical objects that have not appeared previously in your lectures (e.g., you might have a thesis on Brownian motion after a first probability course which hasn't covered the definition of stochastic processes). In such a case, you should expand enough time on the definition to understand how it is motivated, and especially what are the basic fundamental examples (and also possibly counterexamples) of this new notion. You should try to get confortable enough computing with such basic examples to check concretely what the general concepts mean or boil down to in such examples.

(4) If you have to understand an *inequality*, as is often the case in analysis, you should also try to develop a feel for it by "testing" it on special values of the parameters (e.g., specific functions in the case of inequalities between quantities related to functions, such as norms, linear operators, etc).

In an inequality $A \leqslant B$, you can also check what is the "trivial" bound, namely see what inequality $A \leqslant C$ follows from the most straightforward estimates on the terms appearing in A (e.g., just using the triangle inequality, or the Cauchy–Schwarz inequality). This gives you a way to see how far from this the actual inequality is.

Many inequalities involved a number of parameters, and might in fact be "trivial" for certain ranges of some of them (in the sense that the B above would be $\geqslant C$, so that the "trivial" bound $A \leqslant C$ already implies $A \leqslant B$ for these ranges of the parameters). It is useful to identify when this is the case to build some picture of the meaning of the inequality.

(5) When reviewing the literature at the beginning, start going relatively quickly, just surveying what seem the most essential definitions and statements, and make notes of them (e.g. in the skeleton files).

(6) Make notes in particular of further references that may be useful, and follow citations of other papers which seem relevant. Try to locate the original sources of the ideas being discussed, if only to get a basic feeling for the historical picture (this can be enlightnening even if the original papers are too unwieldy to be used as primary references to the change of terminology, etc.)

(7) Be very careful when quoting or using unpublished documents (for instance, results mentioned in a blog post, on Wikipedia, in preprints, on a random place on the internet), since these may well be incorrect or misleading.

    Published sources are not always correct either.

(8) Many people recommend to *write the introduction last*, only after the rest of the manuscript is finalized. There are good reasons for this, but in practice, it can be useful to start recording in writing some of the more general and background information and "philosophical" points as they evolve, and not wait until the last minute. Indeed, waiting too long can have the effect that you are tired of the subject, or bored, or don't have enough time to properly take care of the introduction. This is a problem because this is the first thing that people will read, and often the only part they will read carefully (or at all).

(9) Similarly, don't wait too long before reviewing and correcting or clarifying what you have already written, or for adding proper references if this was not done before. This is again for a question of time and energy.

(10) It is often extremely helpful to include worked-out examples, with concrete computations (do not hesitate to use computers to do such computations when it is possible). This can be extremely helpful for the reader, and enlightening for you.

(11) Build a graph (in your mind or on paper) of the important parts of the theory and connected fields, with links between related parts. This can include "off-topic" aspects which won't be covered in detail, but which are related to the subject.

(12) When writing down a detailed proof, keep in mind two important goals of a proof:

    (a) It should convince you or the reader that the theorem is correct.

    (b) It should help understand *why* the result is true, and possibly how the proof was found.

    The original proofs or those in textbooks are often lacking especially for the second part.

(13) Think carefully about notation. In particular, do not hesitate to change the notation you find in papers or books, so that it becomes more consistent or readable. Changing notation also forces a minimal level of understanding as one reads other texts.

(14) Split proofs in independent and manageable steps, not only for presentation, but also for thinking and understanding long arguments (finding and formulating such a decomposition is also a step towards understanding a proof, isolating parts which maybe are more difficult than others, etc).

(15) For the most important theorems, try to find a good intuitive understanding of why they should be true. For instance, in many problems of analysis, one can "formally" prove many statements by assuming that one can exchange limits at will (for instance, exchange sums and integrals), so the question becomes to justify these steps. Even if this can be a very difficult step, it provides insight on the mechanism involved.

(16) In many cases, proofs are complicated by technical details due to the generality of the statement. It can be enlightening to first go through a proof (or simply certain steps) with simplified assumptions (for instance, assuming that certain functions are smooth, or dealing with some arithmetic problems by assuming that the integers that appear are either equal or coprime). This helps also isolating the real difficulties.

The philosophy is that, in many cases, if one cannot deal with the simplified setting, then there is little chance to be able to handle the most general case. (E.g., if one cannot understand the distribution of integers in a certain sequence, then will not be able to understand the distribution of primes in that sequence).

(17) Try to build your *own* intuition about your topic (or mathematics in general). The insights of other people can be very useful, but your own way of thinking or strengths may be slightly different and lead to different points of view.

For instance, if you like to think in probabilistic terms, many arguments involving measure theory might become more transparent if interpreted from this perspective.

(18) Take time to think by yourself, and try to ask yourself your own questions about the material. These questions will often, at first, be rather naive, but with patience, they help building your own perspective.

(19) Try to bring yourself to the position where you can at least sketch the main ideas of the most important proofs without looking at any notes: you should understand the material enough that the underlying logic (if not the precise details of computations, etc) is clearly present in your mind.

(20) When you cite a paper, either basic background references or specific results you need for a proof, have a look at the original source to make sure that *it says what you claim*. It is often the case that one vaguely remembers a useful resul, but one forgets an assumption that turns out to be important; or one has "heard" that X (often a famous mathematician) proved Y in paper Z, when in fact the history is more complicated, and the first proof was due to a less famous person.

(21) In proofs, do not say that a certain mathematical fact is "unfortunate". Mathematics is the way it is – it is not unfortunate that one cannot solve the quintic equation by radicals. (It can be unfortunate that one doesn't see how to prove a certain result, however.)

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