Due Date: 27 Oct
Commutative Algebra: Fall 2023
Exercise sheet 5

Problem 1. Let $R = \mathbb{C}[x]$ and $R' = \mathbb{C}[x,y]/(y^2 - y, xy)$. Show that $R \subset R'$ is an integral extension with R normal, but does not satisfy the going down property by studying the ideal $(y-1)$ in R'. Why doesn't this contradict the going down theorem?

Problem 2. Let $R' = \mathbb{C}[x,y]$. Let R be the $\mathbb{C}$-subalgebra generated by $x(1-x), y, xy$. Show that $R \subset R'$ is an integral extension of integral domains, but it does not satisfy the going down property. Hints:

1. Find the contraction $\mathfrak{q}$ of the ideal $(1-x, y) \subset R'$ to R.
2. Find the contraction $\mathfrak{p}$ of the ideal $(x) \subset R'$ to R. Show that $\mathfrak{p} \subset \mathfrak{q}$.
3. Show that there is no prime inside of $(1-x, y)$ contracting to contracting to $\mathfrak{p}$.

Why does this not contradict the going down theorem?

Problem 3. Do Gathmann exercise 10.4 and prove Noether Normalization in the finite field case.