Due Date: 24 Nov
Commutative Algebra: Fall 2023
Exercise sheet 9

Problem 1. Compute $\dim \mathbb{C}[x,y]/(y^2-x^3-x^2)$ and $\dim_{\mathbb{C}} \mathfrak{m}/\mathfrak{m}^2$ where $\mathfrak{m}$ is the maximal ideal (x,y) in $\mathbb{C}[x,y]/(y^2-x^3-x^2)$. Compute $\dim_{\mathbb{C}} \mathfrak{n}/\mathfrak{n}^2$ for any other maximal ideal $\mathfrak{n}$.

Problem 2. Prove that a module M over a ring R is Noetherian and Artinian if and only if it has finite length over R.

Problem 3. Suppose that M is a module over a ring R and M has a finite composition series. Show that any two composition series for a module M over R have the same successive quotients, up to reordering.