LECTURE 1 EXERCISES

Exercise 1. Let $T = \mathrm{Spec} A$, with $A := \mathbb{C}[X_1, X_1^{-1}, \cdots, X_n, X_n^{-1}]$ be an algebraic torus. Show that the multiplication map $m : T \times T \to T$ corresponds to the comultiplication map

$$A \to A \otimes A$$
$$X_i \mapsto X_i \otimes X_i$$

the inversion map to

$$A \to A$$
$$X_i \to X_i^{-1}$$

and the identity to

$$A \to \mathbb{C}$$
$$X_i \to 1$$

Exercise 2. Suppose B is a ring over $\mathbb{C}$, $X = \operatorname{Spec} B$, and $T = \operatorname{Spec} A$ is a torus acting on X, $\alpha : T \times X \to X$, so that $\alpha(m(a, b), x) = \alpha(a, \alpha(b, x))$, and $\alpha(1, x) = x$. How can you phrase the data α in terms of the rings A, B?

Exercise 3. Does the variety $X^2 + Y^2 = Z^2$ (i.e. $\operatorname{Spec} \mathbb{C}[X, Y, Z]/(X^2 + Y^2 - Z^2)$) admit the structure of a toric variety?

Exercise 4. Let N be a lattice, with an isomorphism $\phi : N \to \mathbb{Z}^n$. Show that ϕ induces an isomorphism $N \otimes_{\mathbb{Z}} \mathbb{R} \to \mathbb{R}^n$

Exercise 5. Let σ be the 0 cone in $N \cong \mathbb{Z}^n$. Compute the corresponding variety.

Exercise 6. Let N be $\mathbb{Z}^2$. Compute the variety associated to the cone

- $\sigma = \mathrm{Cone}((1, 0), (1, 3))$.
- $\sigma = \mathrm{Cone}((1, 0), (3, 1))$

Do you see the difference?

Exercise 7. Let σ be the cone generated by $(1, 0, 0), (1, 1, 0), (1, 0, 1)$ and $(1, 1, 1)$ in $N = \mathbb{Z}^3$. Find the corresponding variety.

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