LECTURE 3 EXERCISES

(1) Show that homomorphisms of $\mathbb{C}*$ to itself, as algebraic groups, are isomorphic to the integers:

$$\mathrm{Hom}_{\text{alg. gp}}(\mathbb{C}^*, \mathbb{C}^*) = \mathbb{Z}$$

(Hint: such homomorphisms are homomorphisms of $\mathbb{C}[x, x^{-1}]$ that respect the comultiplication).

(2) Show that

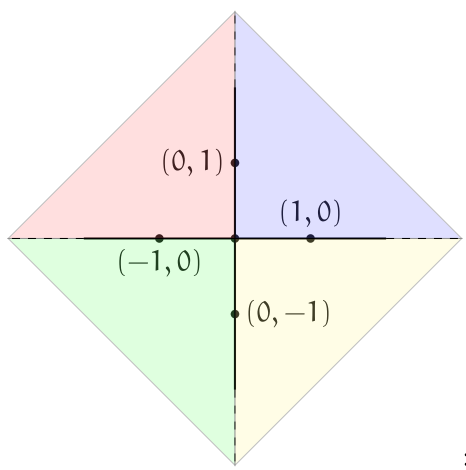
$$\mathrm{Hom}_{\text{Scheme}}(\mathrm{Spec}\, A, \mathrm{Spec}\, \mathbb{C}[S_\sigma]) = \mathrm{Hom}_{\text{Mon}}(S_\sigma, A)$$

where A is considered to be a monoid under multiplication.

(3) Find the fan of the toric variety

$$\mathrm{Spec}\, \mathbb{C}[x, y, z]/xz = y^3$$

(4) Can the following be the fan of $\mathbb{P}^2$? (bonus if you can come up with a 10 second explanation)



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(5) Let $N = \mathbb{Z}^3$, $v_1 = (1, 0, 0), v_2 = (0, 1, 0), v_3 = (0, 0, 1), u = (-1, -1, -1)$. Let Δ be the fan consisting of the four maximal cones $\mathrm{Cone}(v_1, v_2, v_3), \mathrm{Cone}(v_1, v_2, u), \mathrm{Cone}(v_1, v_3, u), \mathrm{Cone}(v_2, v_3, u)$. Compute the fan of the stratum corresponding to the ray $\mathbb{R}_{\geqslant 0} v_1$.