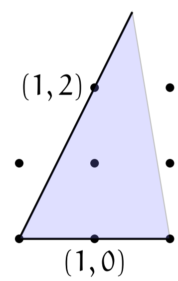
LECTURE 4 EXERCISES

Exercise 1. Let



be a cone in $\mathbb{Z}^2$, and write $X(\sigma, N)$ for the corresponding toric variety. Can you find a toric compactification of this variety? That is, a compact toric variety Y that contains $X(\sigma, N)$ as a dense open subset.

Exercise 2. A map of lattices $p : N \to L$, and fans $F \subset N_{\mathbb{R}}$, $G \subset L_{\mathbb{R}}$ are given. Do these data represent toric morphisms? If so, are they proper? Are they birational?

- Let $p : \mathbb{Z}^2 \to \mathbb{Z}$ be the addition map $(a, b) \to a + b$; $F = \sigma$ is the cone spanned by $(-1, -1)$ and $(1, 0)$, and $G = \tau$ the cone $\mathbb{R}_{\geqslant 0}$.
- p and G as above, and F in $\mathbb{Z}^2$ the fan with cones $\text{Cone}((1, 0), (0, 1))$ and $\text{Cone}((1, 0), (0, -1))$.
- $p : \mathbb{Z} \to \mathbb{Z}$ the inversion map $x \to -x$, and $F = G = \mathbb{R}_{\geqslant 0}$.
- p as above, F the fan with two cones $\mathbb{R}_{\geqslant 0}1, \mathbb{R}_{\geqslant 0}(-1)$, and G as above.
- $p : \mathbb{Z}^2$ to $\mathbb{Z}^2$ the identity, F the fan spanned by the two maximal cones $\text{Cone}((1, 0), (1, 1))$ and $\text{Cone}((1, 1), (0, 1))$, and G the fan $\text{Cone}((1, 0), (0, 1))$.

Exercise 3. Show that any map from a proper toric variety to a toric variety is proper.

Exercise 4. Let $p : N \to L$ be a map of lattices, and $(\sigma, N) \to (\kappa, L)$ a map of affine toric varieties. Classify when it is possible for the map to be proper.