LECTURE 6 EXERCISES

Exercise 1. Do the exercises from the notes on sheaves and cohomology.

Exercise 2. Let $X = X(\Delta, N)$ be a toric variety, D a divisor on X, associated to a fractional ideal I. Show that an action of the torus $T = \operatorname{Spec} \mathbb{C}[M]$ on D is equivalent to the data of an M-grading on I, i.e. a splitting

$$I = \oplus_{m \in M} I_m$$

(Use the fact that multiplication on T corresponds to comultiplication $\mathbb{C}[M] \to \mathbb{C}[M] \otimes \mathbb{C}[M]$ on the level of rings.

Exercise 3. Compute the cohomology groups of the constant sheaf $\underline{\mathbb{Z}}$ on the n-dimensional sphere S^n

$$H^i(S^n, \underline{\mathbb{Z}}) = ?$$

Exercise 4. Compute

$$H^i(\mathbb{P}^n, \underline{\mathbb{Z}})$$

Exercise 5. Let X be a variety, and suppose F is a sheaf with the property that for all $U \subset V$ open,

$$F(U) \to F(V)$$

is surjective. Show that

$$H^i(X, F) = 0$$

for $i > 0$.

Exercise 6. Let X be a topological space.

Definition 1. A resolution of a sheaf F is an exact sequence

$$0 \longrightarrow F \longrightarrow I_0 \longrightarrow I_1 \longrightarrow I_2 \longrightarrow \cdots$$

Suppose that all the sheaves I_i are "acyclic" on X, i.e.

$$H^k(X, I_i) = 0$$

for all $i,k > 0$. Show that

$$H^k(X, F) = \ker\left(H^0(X, I_k) \to H^0(X, I_{k+1})\right) / \operatorname{Im} H^0(X, I_{k-1})$$

(Hint: Use the long exact sequence repeatedly. Try it for resolutions of length $1, 2, 3$ first).