Exercise Sheet 11

1. Let Γ be a group with a normal subgroup $N \triangleleft \Gamma$ such that N and Γ/N satisfy the conclusion of the Markov-Kakutani fixed point theorem. Show that it holds for Γ as well.

2. Show that if Γ has property (F) (Remark VI.24) then it satisfies the conclusions of the MK-fixed point theorem.

3. Determine the extreme points of $M^1([0,1])$.

4. Let $\varphi\colon [0,1] \to [0,1], \varphi(x) := x^2$; determine all the φ-invariant probability measures.

5. In Example VI.26 justify why for $\alpha \notin \mathbb{Q}/\mathbb{Z}$, λ is the unique T_α-invariant probability measure.

6. Let $T \in SL_2(\mathbb{R})$ with real eigenvalues $\{\lambda, \lambda^{-1}\}$ with $\lambda > 1$. Then T induces a homeomorphism of $\mathbb{P}^1(\mathbb{R})$ still denoted T. Classify all T-invariant probablity measures on $\mathbb{P}^1(\mathbb{R})$. Prove that there exists no probability measure on $\mathbb{P}^1(\mathbb{R})$ which is invariant under $\mathrm{SL}_2(\mathbb{Z})$.