Exercise Sheet 12

1. Let $G \times X \to X$ be a countable group G acting on a compact Hausdorff space X by homeomorphisms. Show that the set $M^1(X)^G$ of G-invariant probability measures on X is a weak*-closed, convex subset of $M^1(X)$.

2. Let (X, d) be a compact metric space and $(f_n)_n$ be a sequence of continuous functions converging pointwise to a continuous function $g \colon X \to \mathbb{R}$. Assume

$$f_n(x) \leqslant f_{n+1}(x) \leqslant g(x)$$

   for all $n \geqslant 1$ and $x \in X$. Prove that $(f_n)_n$ converges uniformly to g.

3. Prove Lemma VII.6.