Exercise Sheet 8

1. Prove Proposition V.11 and Corollary V.12.
2. Let V be a finite-dimensional normed space. Show that the weak topology and the norm topology on V coincide.
3. Let X be a set, $\mathscr{F} = \{(\varphi, Y_i) : i \in I\}$ a family of pairs consisting of topological spaces Y_i with a map $\varphi_i \colon X \to Y_i$ and equip X with the initial topology with respect to $\mathscr{F}$. Prove that a sequence $(x_n)_n \in X$ converges to $x \in X$ if and only if $\varphi_i(x_n)$ converges to $\varphi_i(x)$ for all $i \in I$.
4. Show that on $L^2_{\mathrm{loc}}(\mathbb{R})$ (where we take the Lebesgue measure) there is no norm inducing the topology defined in Example V.10.