Exercise Sheet 9

1. Let $T\colon V \to W$ be a bounded linear operator of normed spaces. Prove that the adjoint $T^*\colon W^* \to V^*$ is continuous in the weak*-topology on W^* and V^*.

2. Let V be a normed space. Assume that $f\colon V \to \mathbb{K}$ is a linear form that is continuous w.r.t. the weak topology on V. Show that f is strongly continuous.

3. For each $n \geqslant 1$ define
$$\mu_n := \frac{1}{n}\sum_{i=1}^{n} \delta_{k/n}.$$
What is the weak*-limit of the sequence μ_n as $n \to \infty$?

4. Let V be a normed space. Show that any linear form $\lambda\colon V^* \to \mathbb{K}$ that is continuous with respect to the weak*-topology is of the form $\lambda(f) = f(v)$ for some $v \in V$.

   Hint: The solution is similar to the solution of Exercise 2.