Exercise Sheet 4

To be handed in until October 18

1. Helicoid and catenoid are locally isometric

(a) Find a local isometry

$$\varphi : \text{helicoid} \to \text{catenoid}.$$

(b) Verify that φ preserves K. What does it do to the principle curvatures and principle directions?

(c*) Show that there is a continuous family of minimal surfaces deforming the helicoid into the catenoid.

2. More on isometries

A local isometry between surfaces in $\mathbb{R}^3$ preserves the Gauss curvature K but normally not k_1, k_2, H, or the principal directions of curvature. So the situation in exercise 1 was special in this respect.

(a) Compute K, k_1, k_2, H of a cone.

(b) Show that the cone (minus the vertex) is locally isometric to the plane.

(c) Is there a global isometry between the cone (minus the vertex) and the plane minus a point?

3. The Pseudosphere

The pseudosphere is the surface of revolution for the curve $\gamma : \mathbb{R} \to \mathbb{R}^2$ parametrized by

$$t \mapsto (t - \tanh t, \operatorname{sech} t),$$

where $\tanh t = \frac{\sinh t}{\cosh t}$ and $\operatorname{sech} t = \frac{1}{\cosh t}$. Compute K for the pseudosphere.

4. Shortest path in $\mathbb{R}^n$

Prove that a straight line in $\mathbb{R}^n$ is the shortest path between two given points.