The most important exercises are marked with an asterisk *.

***10.1.** Let (M, ω) and (M', ω') be a symplectic manifolds of the same dimension. Let $\psi\colon M \to M'$ be a smooth map and consider its graph

$$\Gamma_\psi \coloneqq \{(x, \psi(x)) \,|\, x \in M\} \subset (M \times M', \omega \oplus (-\omega')).$$

Show that ψ is symplectic (i.e. $\psi^*\omega' = \omega$) if and only if Γ_ψ is a Lagrangian submanifold.

***10.2.** Let Q be a smooth manifold and $S \subset Q$ a submanifold. Prove that the annihilator

$$TS^0 \coloneqq \{(x, \xi) \in T^*Q \,|\, x \in S, \xi|_{T_xS} \equiv 0\}$$

is a Lagrangian submanifold of T^*Q with respect to the canonical symplectic form.

10.3.

(a) Let X be a smooth manifold. Show that any diffeomorphism $\psi\colon X \to X$ lifts to a symplectomorphism

$$\Psi\colon T^*X \to T^*X$$

by the formula

$$\Psi(x, \xi) = (\psi(x), \xi \circ (\mathrm{d}\psi_x)^{-1}).$$

(b) Consider $(\mathbb{C}^2 \cong \mathbb{R}^4, \omega_{\mathrm{std}})$ and the submanifold

$$L_{\mathrm{Ch}} \coloneqq \left\{ \begin{pmatrix} (e^s + ie^{-s}t)\cos(\theta) \\ (e^s + ie^{-s}t)\sin(\theta) \end{pmatrix} \middle|\, \theta, s, t \in \mathbb{R}, s^2 + t^2 = 1 \right\}.$$

Show that this is a Lagrangian submanifold. It is called *Chekanov torus*.

Hint: Write L_{Ch} as the image of a Lagrangian in $T^*S^1 \times T^*\mathbb{R}$ under a symplectomorphism.