The most important exercises are marked with an asterisk *.

***11.1.** Let (M, ω) be a closed symplectic manifold.

(a) Let X be a symplectic vector field, i.e. a vector field satisfying $\mathrm{d}\iota_X\omega = 0$. Let $\psi \in \mathrm{Symp}_0(M, \omega)$. Show that $\psi^*X - X$ is a Hamiltonian vector field.

(b) Suppose the smooth isotopies $\varphi_t, \psi_t \in \mathrm{Diff}(M)$ are generated by the vector fields X_t and Y_t. Compute the vector field that generates $\psi_t \circ \varphi_t$.

(c) Deduce that

$$\mathrm{Flux}\colon \widetilde{\mathrm{Symp}_0}(M, \omega) \to \mathrm{H}^1(M, \mathbb{R})$$

is a homomorphism of groups.

***11.2.** Let (M, ω) be a closed symplectic manifold. Show that Flux is surjective.

11.3. Let (M, ω) be a closed symplectic manifold. Let ψ_t and φ_t be symplectic isotopies with $\psi_0 = \varphi_0 = \mathrm{id}$. Consider the *juxtaposition* of ψ_t and φ_t, which is defined by

$$\chi_t = \begin{cases} \varphi_{2t}, & 0 \le t \le \frac{1}{2}, \\ \psi_{2t-1} \circ \varphi_1, & \frac{1}{2} \le t \le 1. \end{cases}$$

Prove that χ_t and $\psi_t \circ \varphi_t$ represent the same element in $\widetilde{\mathrm{Symp}_0}(M, \omega)$. Use it to give a different proof for Flux being a homomorphism.