The most important exercises are marked with an asterisk *.

12.1. Let (M,ω) be a closed symplectic manifold. Consider

$$M \times M^- := \Big(M \times M,\ \omega \oplus (-\omega)\Big).$$

Let $j_\Delta\colon M \to M \times M^-$ denote the diagonal inclusion $j(x) = (x,x)$.

Let ψ_t, $t \in [0,1]$, be a symplectic isotopy with $\psi_0 = \mathrm{id}$. Show that

$$\mathrm{Flux}(\{\psi_t\}) = -j_\Delta^* \,\mathrm{Flux}(\{\mathrm{id} \times \psi_t\}).$$

***12.2.** Let $\chi\colon M \to M$ be a symplectomorphism on a closed symplectic manifold (M,ω) and let ψ_t, $t \in [0,1]$, be a symplectic isotopy with $\psi_0 = \mathrm{id}$. Show that

$$\mathrm{Flux}(\{\chi^{-1} \circ \psi_t \circ \chi\}) = \chi^*(\mathrm{Flux}(\{\psi_t\}).$$

***12.3.** Consider the exact symplectic manifold $(T^*Q, \mathrm{d}\alpha)$, where $\alpha \in \Omega^1(Q)$ is the canonical 1-form on M. If $\sigma \in \Omega^1(Q)$ is a closed 1-form, there is an associated diffeomorphism $\nu_\sigma\colon T^*Q \to T^*Q$ defined by

$$\nu_\sigma(q,\xi) = (q, \xi + \sigma_q).$$

(a) Prove that

$$\nu_\sigma^*\alpha - \alpha = \pi^*\sigma,$$

where $\pi\colon T^*Q \to Q$ denotes the canonical projection.

(b) Prove that ν_σ is a symplectomorphism.

(c) Prove that ν_σ is a Hamiltonian diffeomorphism if and only if σ is exact.