

This is a sample exam question.

**\*13.1. Hamiltonian diffeomorphisms**

Let  $(M, \omega)$  be a closed symplectic manifold.

- (a) (3 points) What is a Hamiltonian diffeomorphism on  $M$ ?
- (b) (4 points) Let  $H: [0, 1] \times M \rightarrow \mathbb{R}$  be a smooth Hamiltonian function and  $\psi_t^H$  the corresponding Hamiltonian flow. Let  $\chi$  be a symplectomorphism on  $M$ . Show that  $\chi^{-1}\psi_t^H\chi$  is generated by  $H_t \circ \chi$ .
- (c) (4 points) Consider the 2-sphere  $S^2 \subset \mathbb{R}^3$  endowed with the standard symplectic form given by

$$\omega_x(v, w) = x \cdot (v \times w),$$

for all  $x \in S^2$  and  $v, w \in T_x S^2 = \{v \in \mathbb{R}^3 \mid x \cdot v = 0\}$ . Let  $H: S^2 \rightarrow \mathbb{R}$  be the autonomous Hamiltonian function given by

$$(x_1, x_2, x_3) \mapsto x_3.$$

Compute the corresponding Hamiltonian flow  $\psi_t^H$ ,  $t \in \mathbb{R}$ .