Preparation for Probability Theory 2023 ETHZ exam

* The exam will contain one or several of the following qustions:

– Any one of the "Exercise covered during the exercise class" from Sheets 2, 7, 10, 11, 12, 13

– State the Dynkin Lemma.

– State the Borel–Cantelli lemmas.

– Give the definition of the law of a random variable.

– State the transfer theorem.

– Show that if X and Y are two independent real-valued random variables with densities then $\mathbb{P}(X = Y) = 0$.

– State the Kolmogorov two-series theorem.

– State and prove Kolmogorov's three-series theorem (using Kolmogorov's two-series theorem without proof).

– State the strong law of large numbers.

– Let $(M_n)_{n\geq 0}$ be a $(\mathcal{F}_n)_{n\geq 0}$ martingale and T a $(\mathcal{F}_n)_{n\geq 0}$-stopping time. Show that $(M_{n\wedge T})_{n\geq 0}$ is a $(\mathcal{F}_n)_{n\geq 0}$ martingale (Lemma page 4 in Chapter 4 of the lecture notes).

– For $X \in L^1(\Omega, \mathcal{F}, \mathbb{P})$ and $(\mathcal{A}_i)_{i\in I}$ a collection of σ-fields included in $\mathcal{F}$, show that the family of random variables $(\mathbb{E}[X \mid \mathcal{A}_i])_{i\in I}$ is uniformly integrable (Proposition page 2 in Chapter 5 of the lecture notes).

– State the optional stopping theorem.

– State the convergence theorem concerning martingales bounded in L^p, $p > 1$.

– State the Portemanteau theorem.

– State Lévy's theorem.

– State the central limit theorem.

WARNING. Be sure to properly state all the required assumptions when asked to state a result.