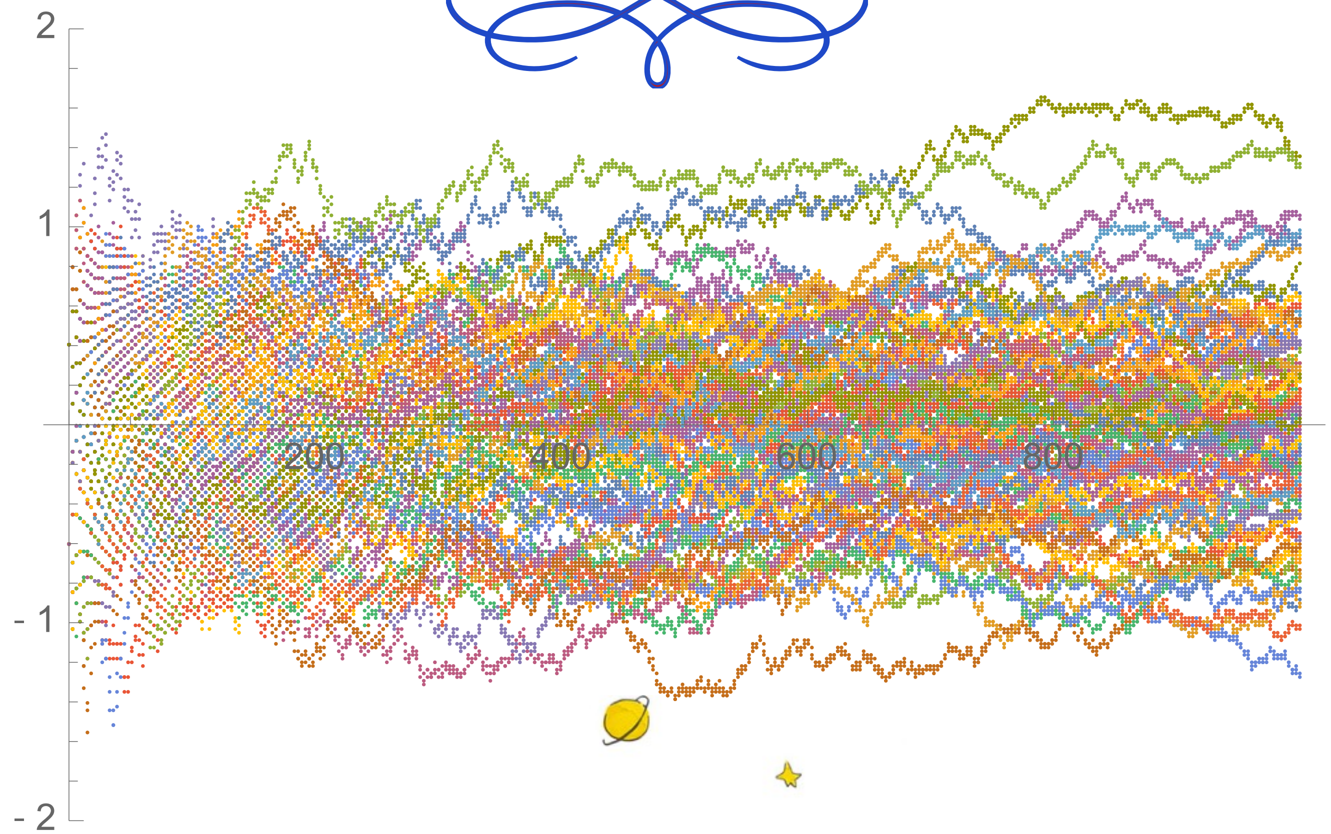
Predicting the Unpredictable : Structure in Randomness



Igor Kortchemski – ETH Zürich

Probability Theory Autumn 2023

I. A DREAM

A dream...

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And in answer he repeated, very slowly, as if he were speaking of a matter of great consequence:

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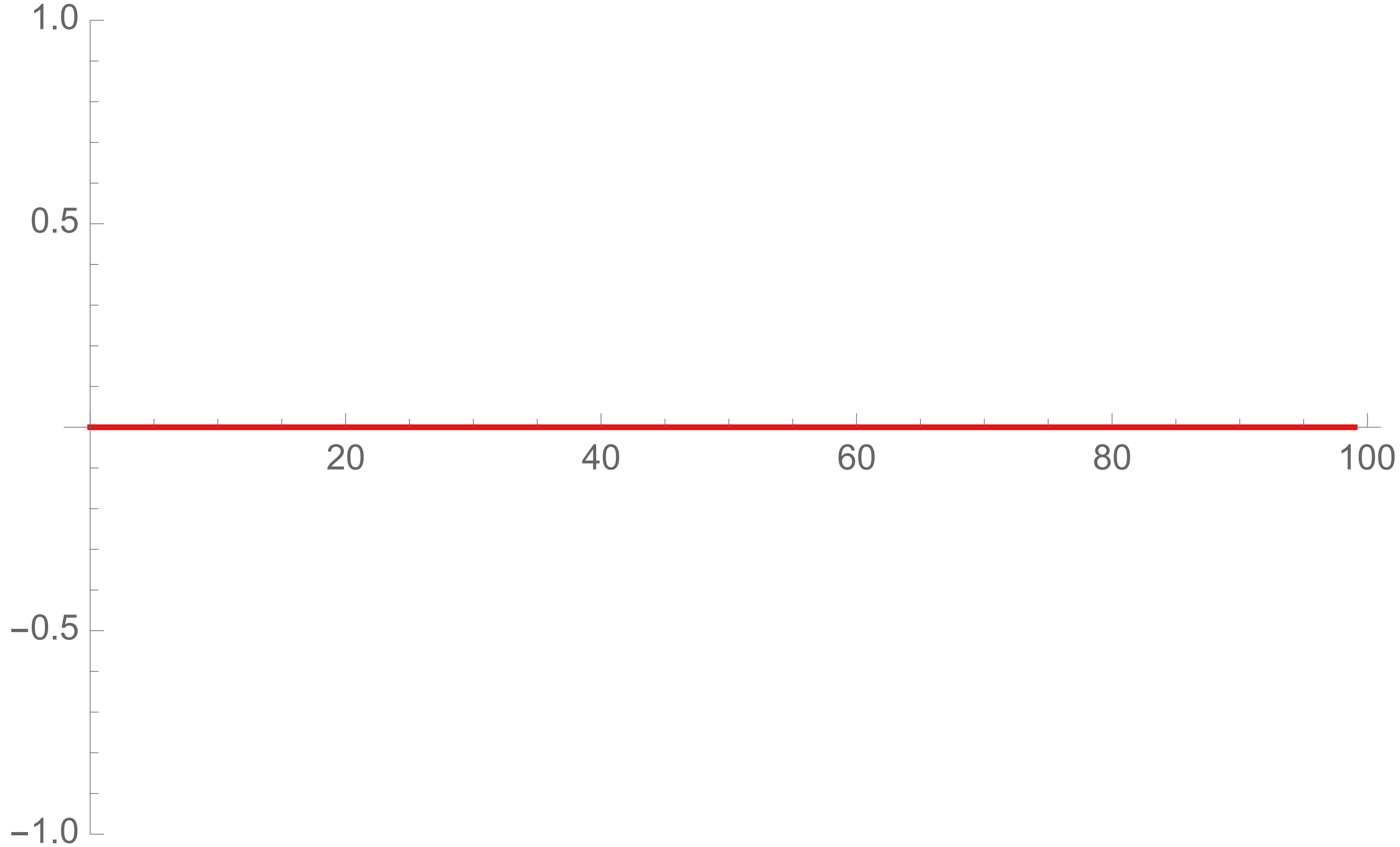

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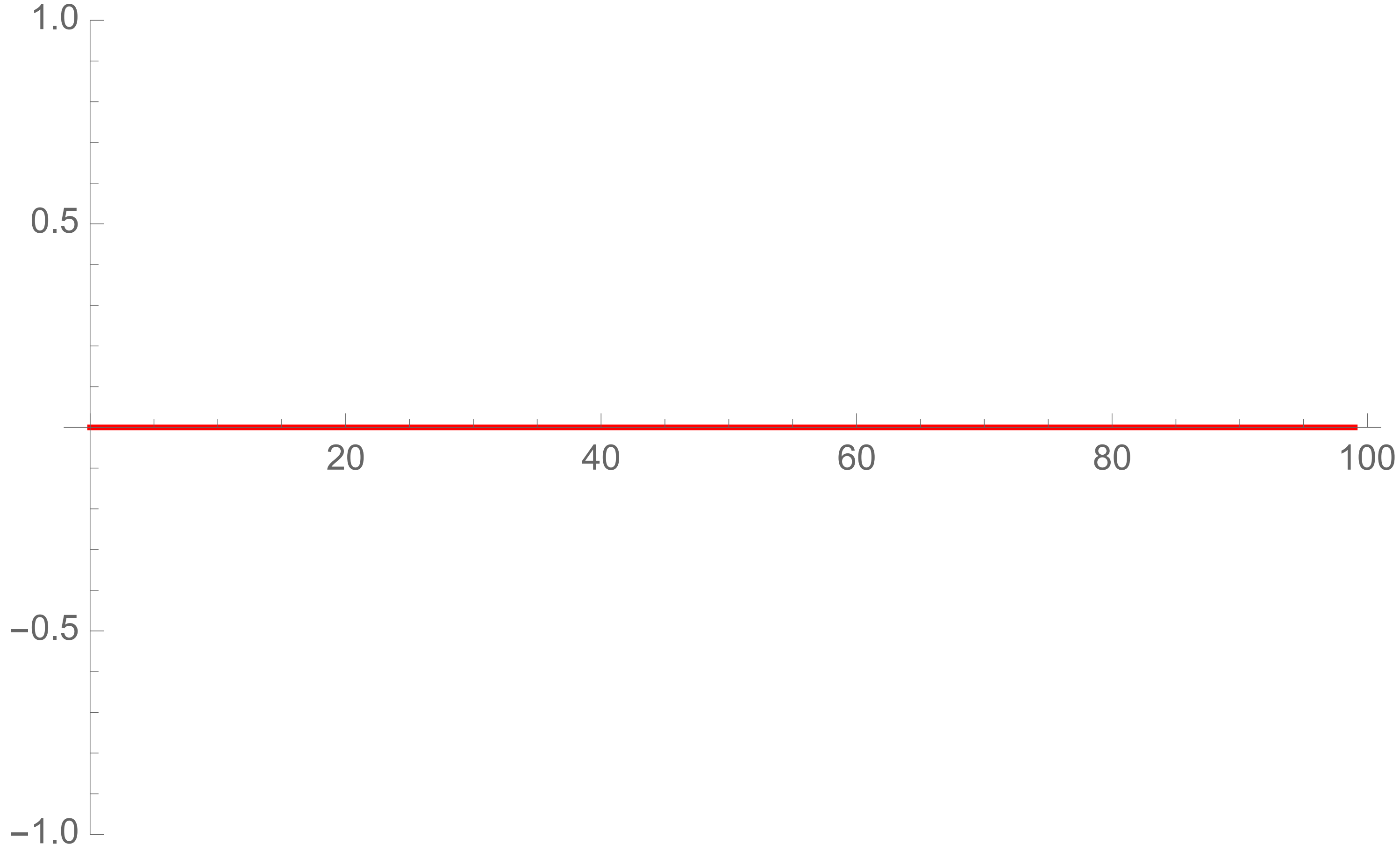

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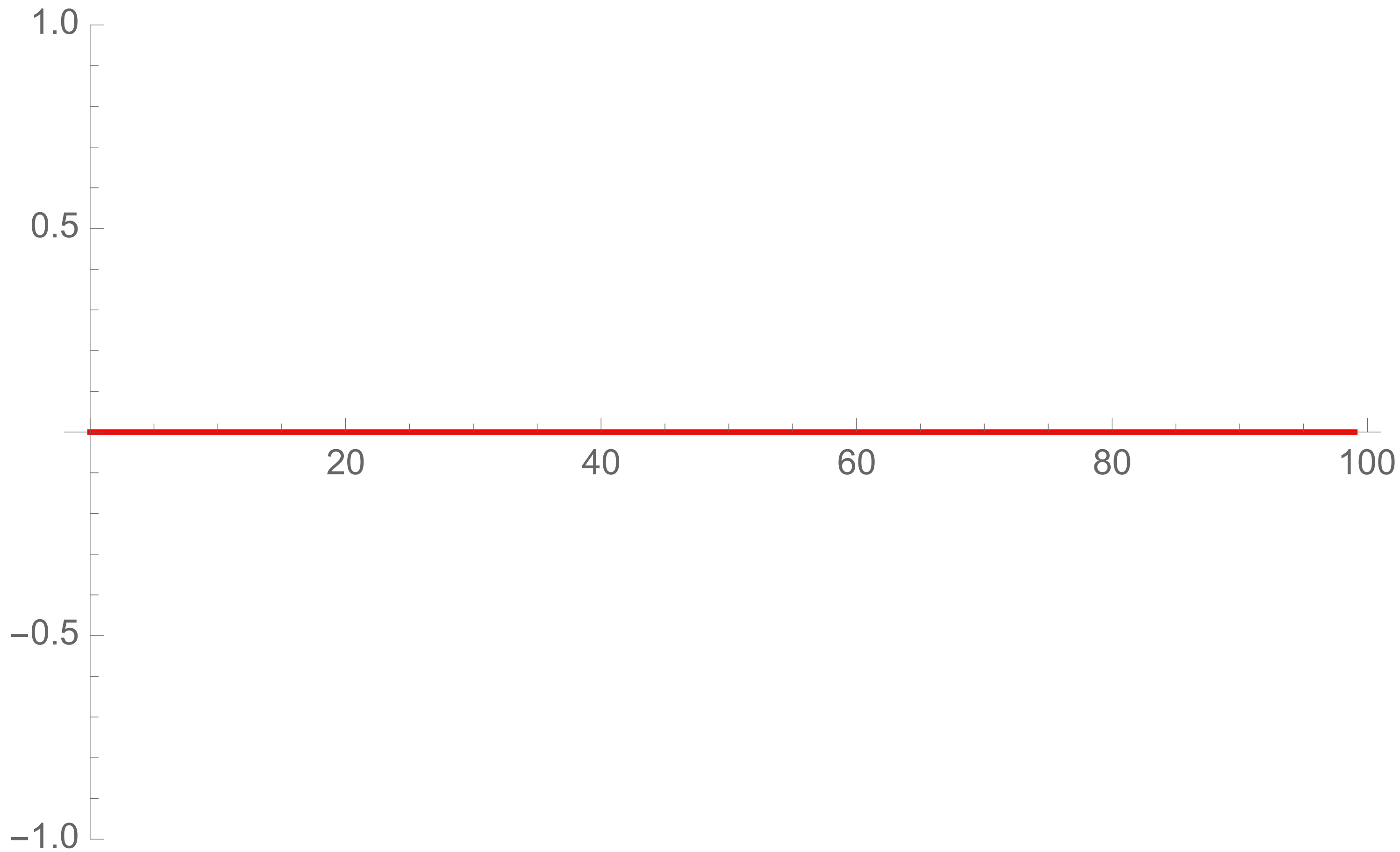

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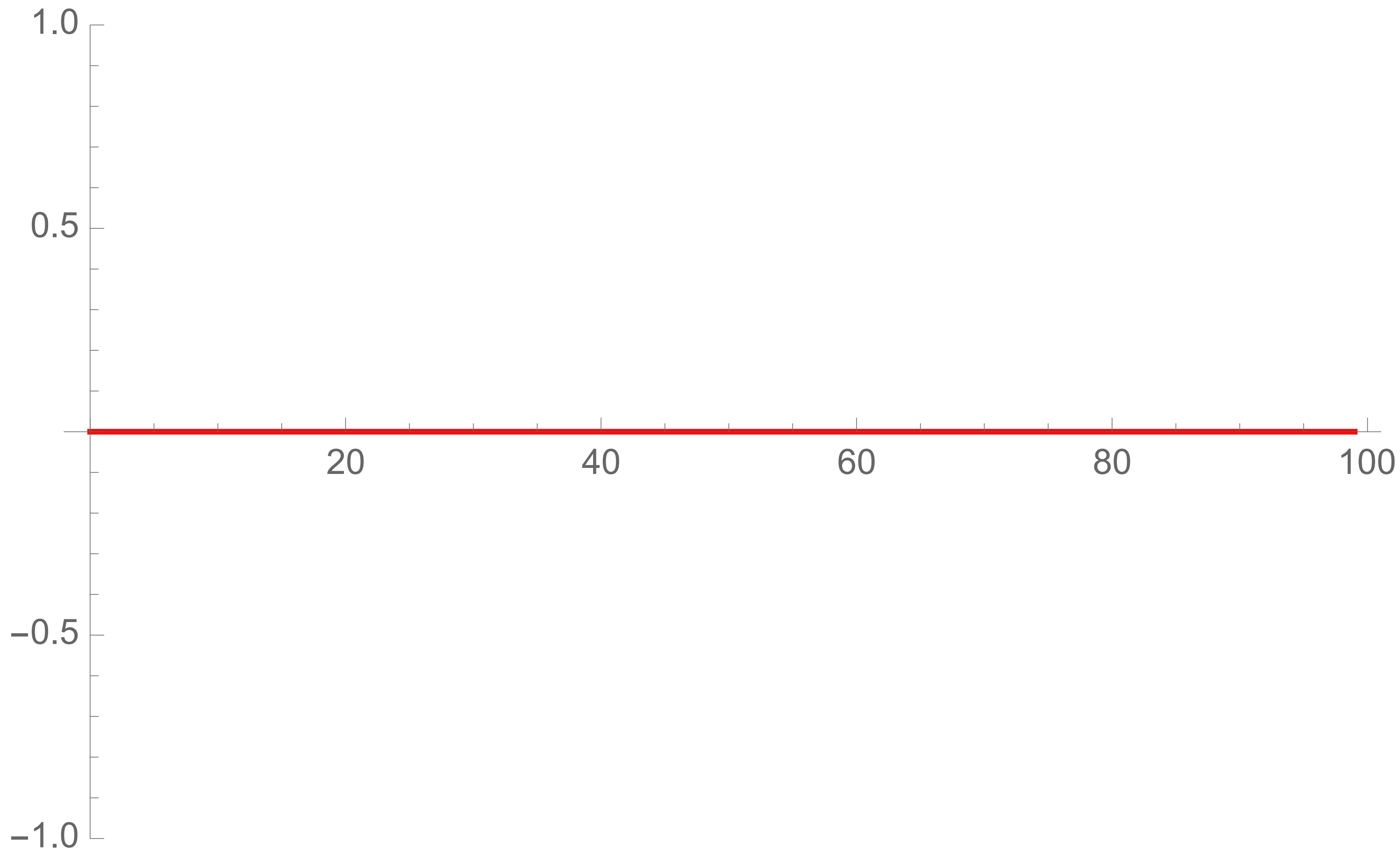

"No, no, no! I do not want a function that doesn't wiggle. I need a random function. Draw me a function at random."

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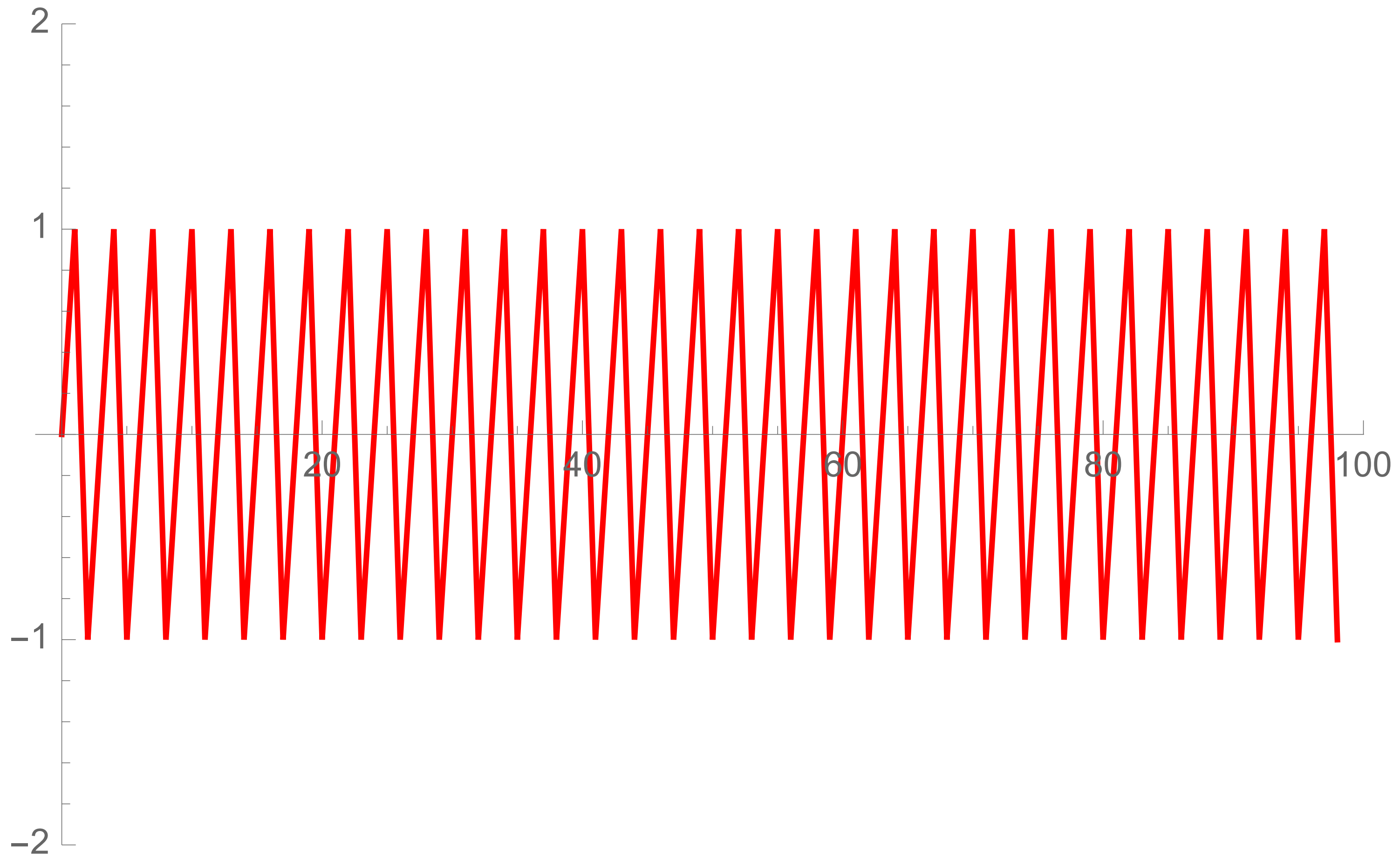

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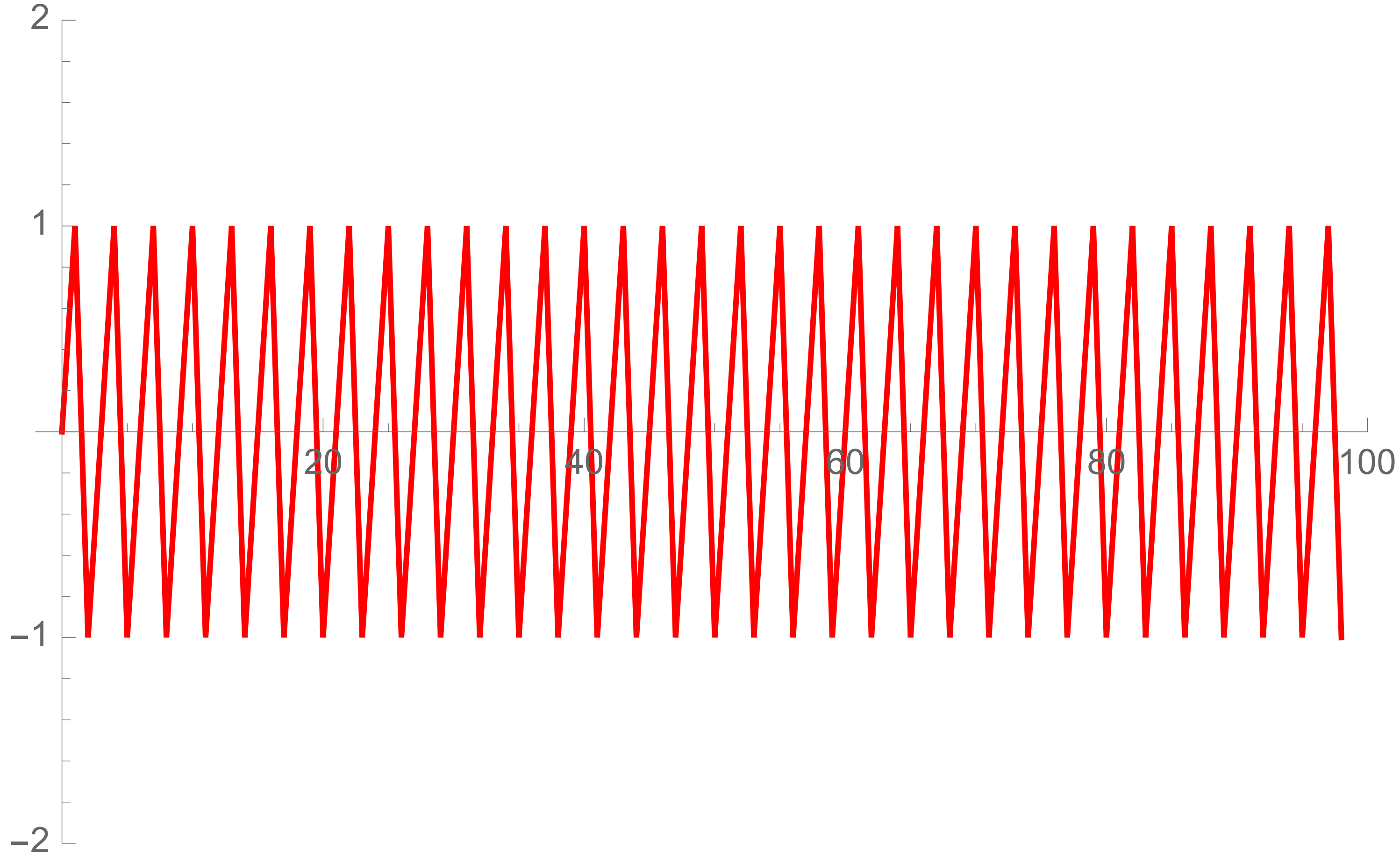

He looked at it carefully, then he said:

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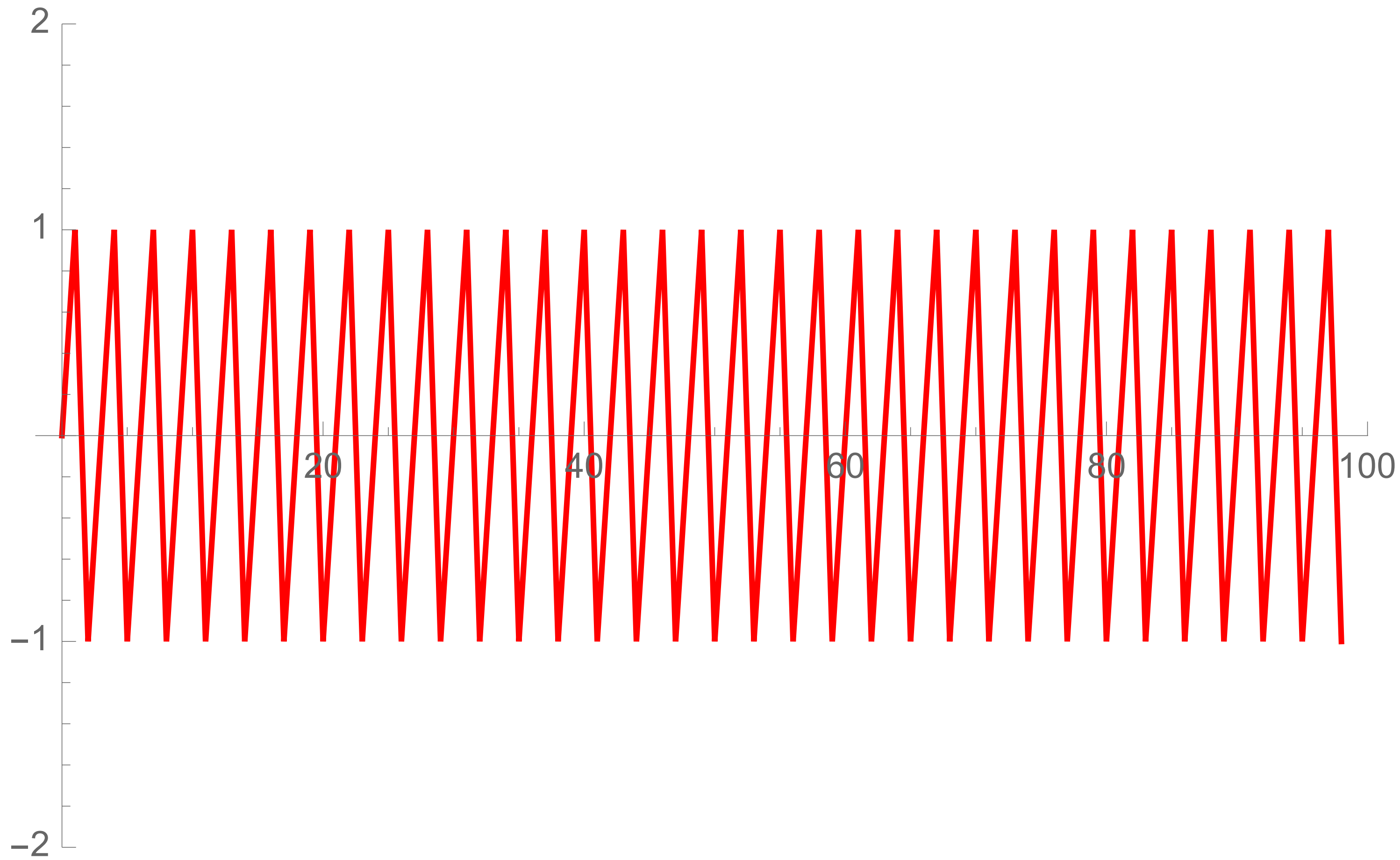

He looked at it carefully, then he said:
"No. This one is predictable.

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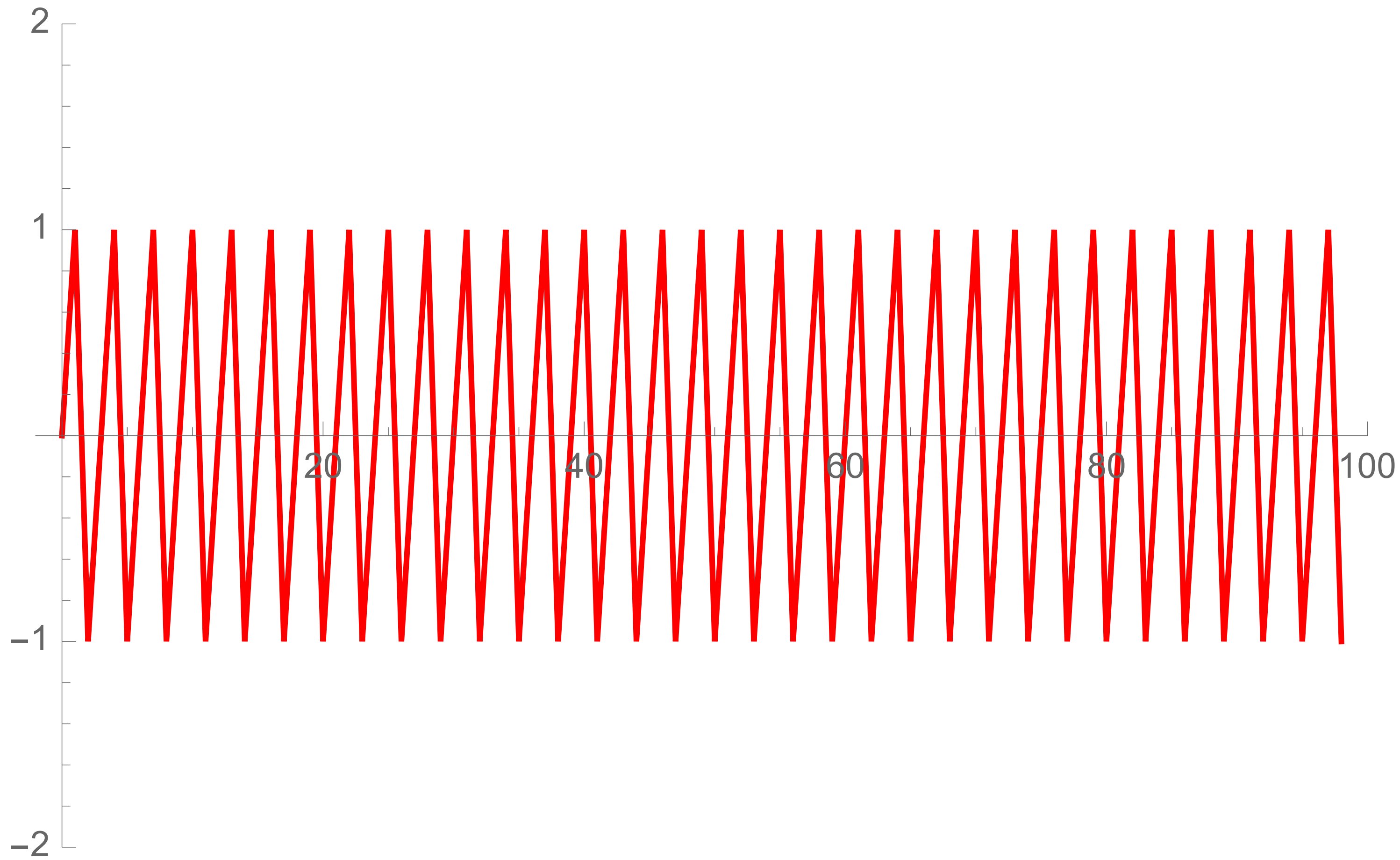

He looked at it carefully, then he said:
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For an integer $n \geqslant 1$, define

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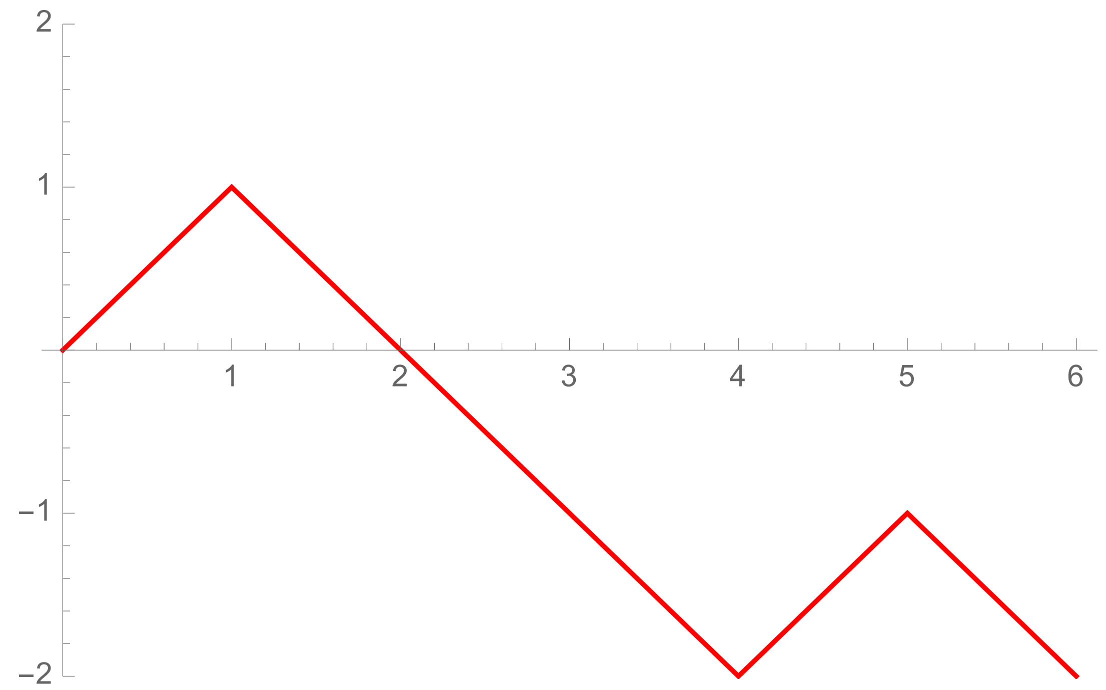

Figure: The path $(0, 1, 0, -1, -2, -1, -2)$.

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And I threw out an explanation with it:

"This is a finite set, with 2^n elements. Choose a function uniformly at random inside. This is the function you asked for."

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I was very surprised to see a light break over the face of my young judge:

"That is exactly the way I wanted it!"

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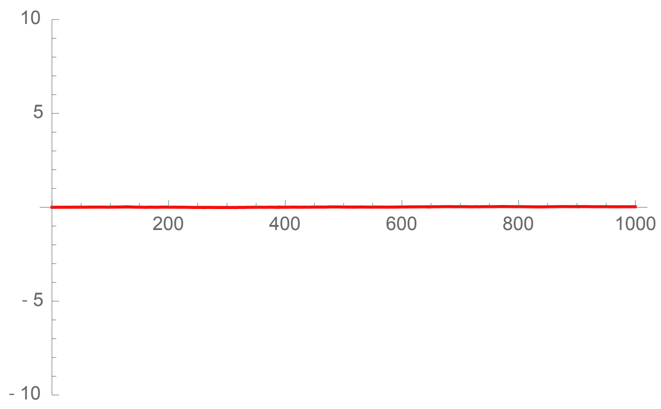

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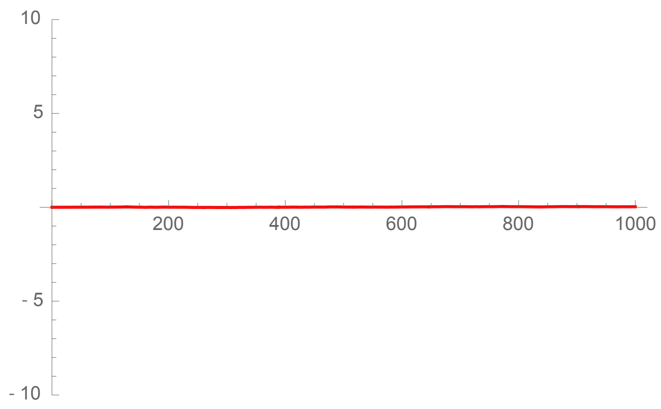

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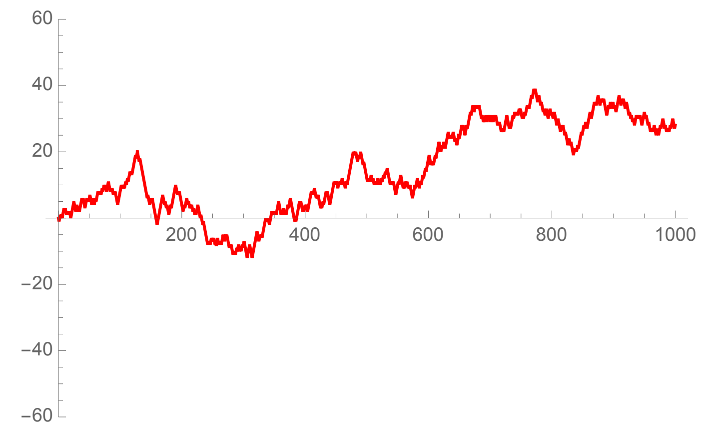

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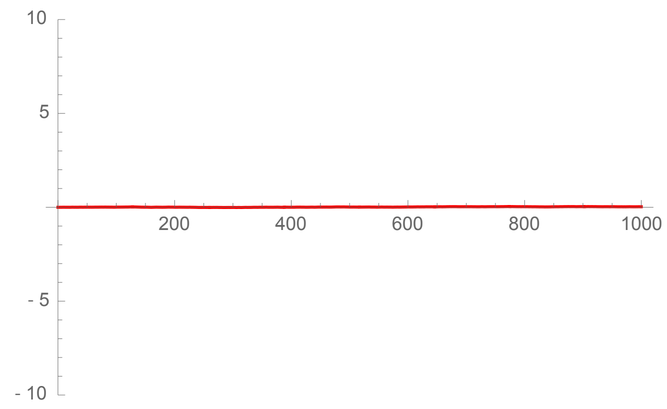

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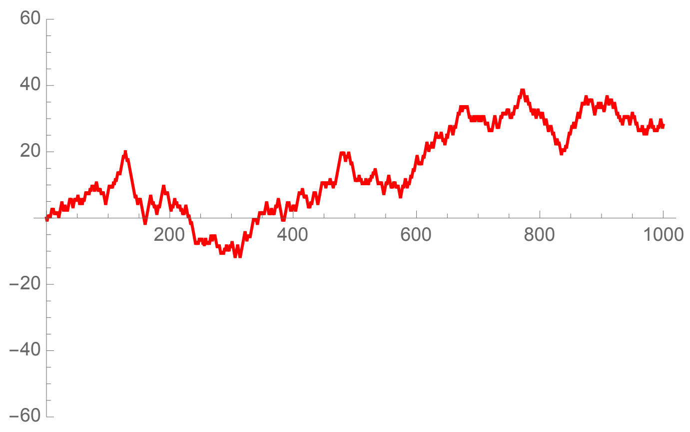

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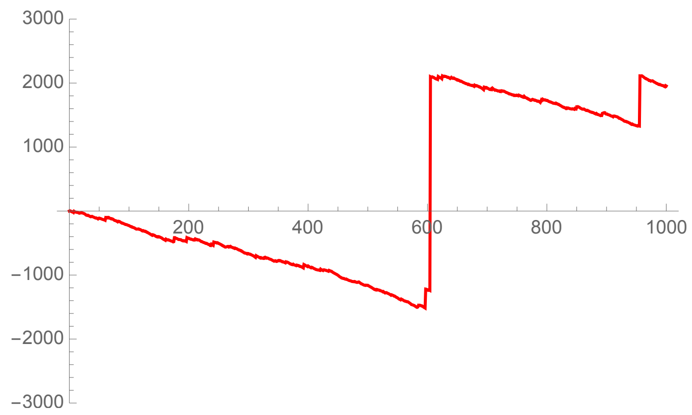

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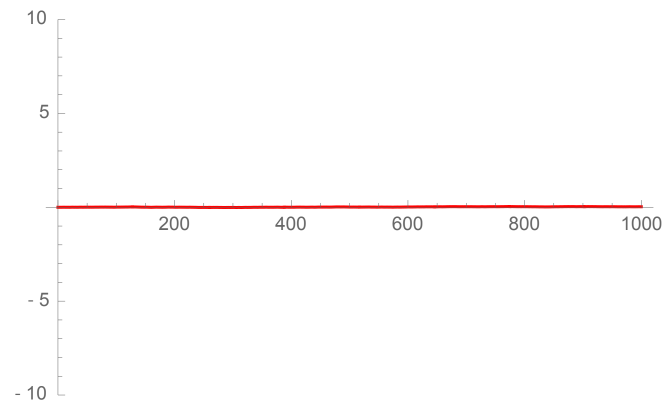

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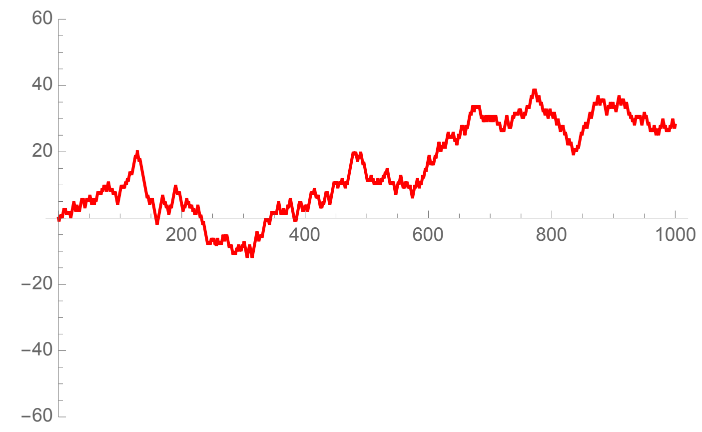

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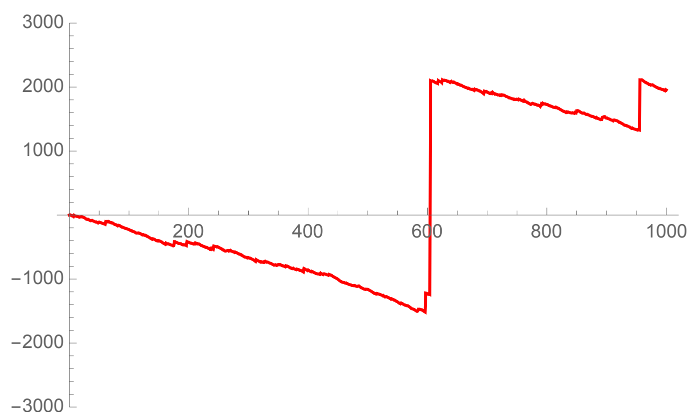

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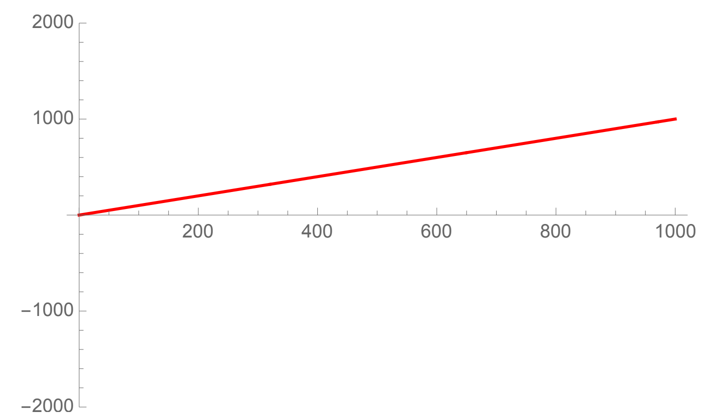

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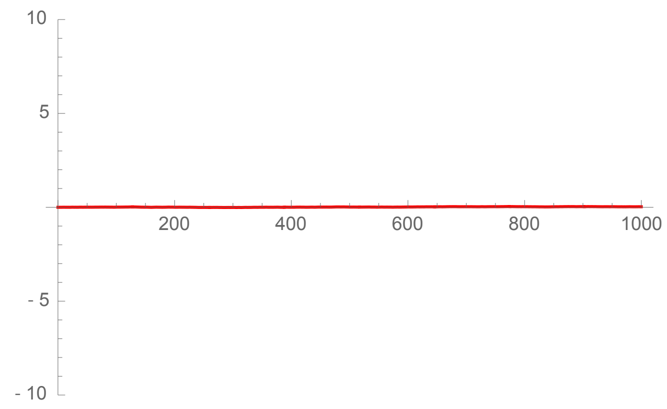

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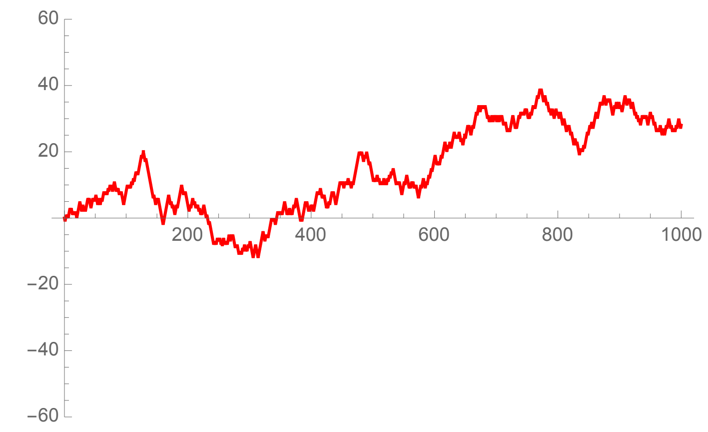

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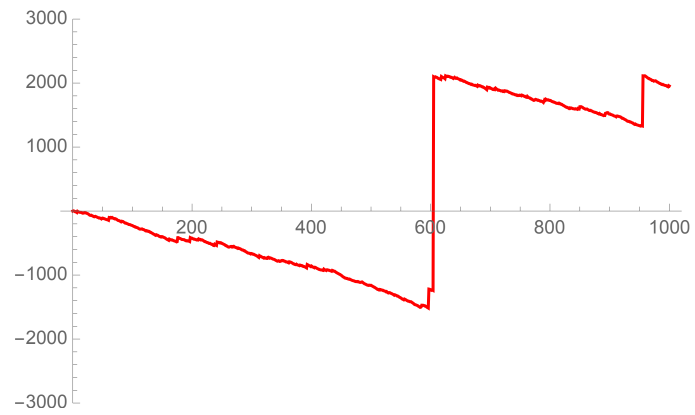

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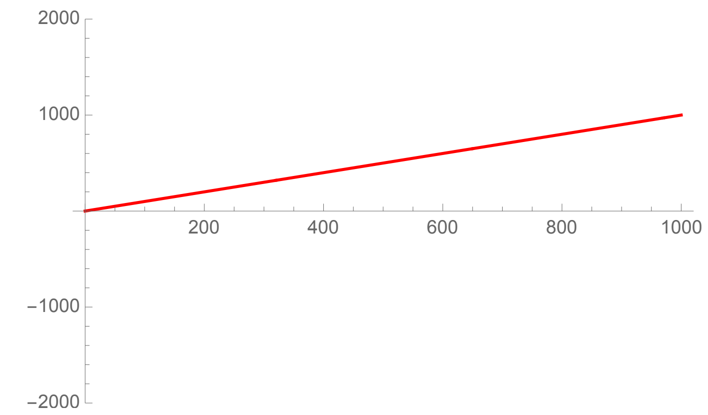

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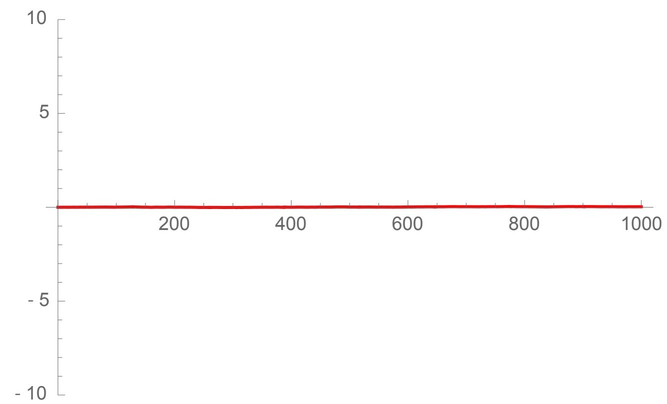

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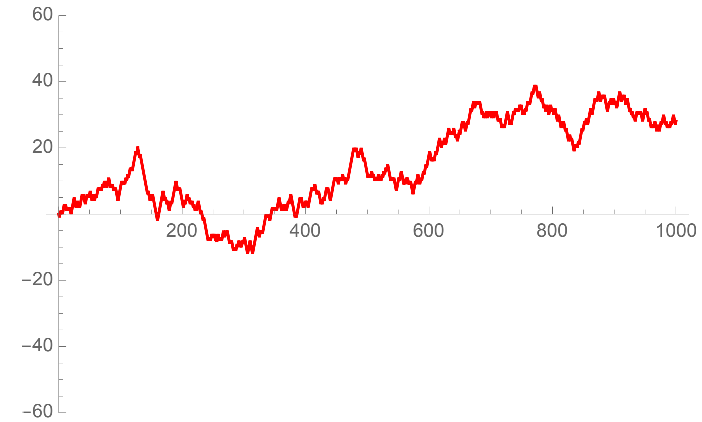

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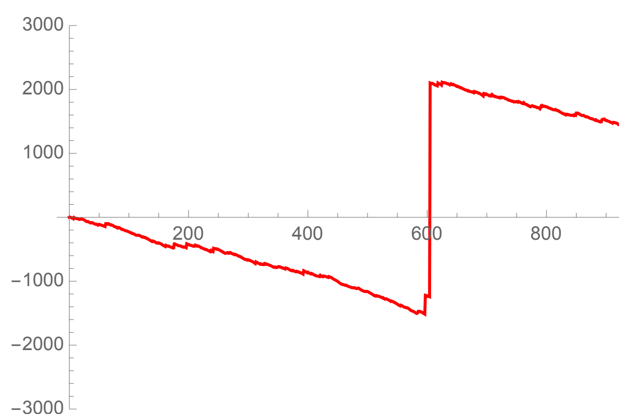

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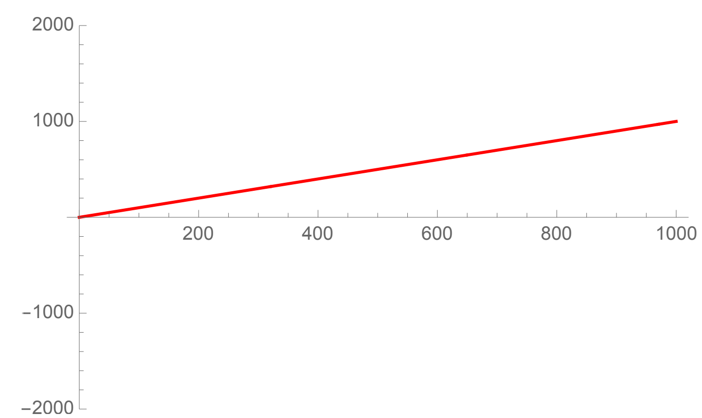

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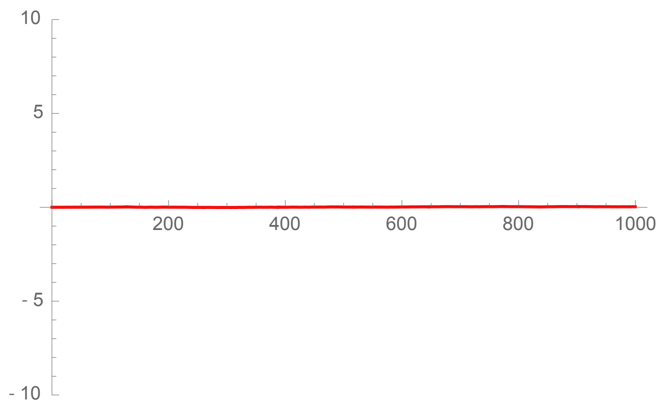

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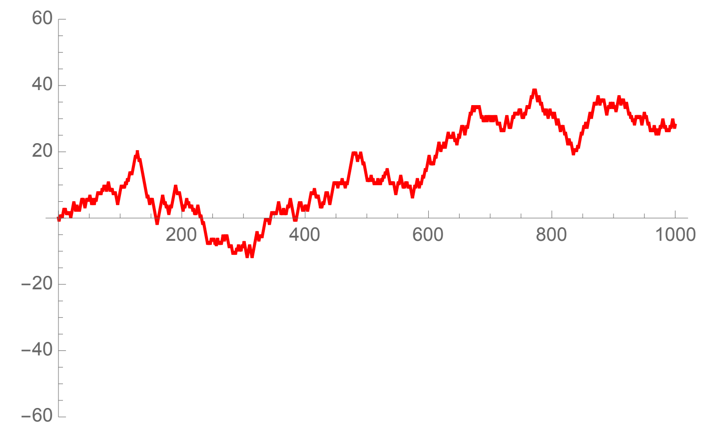

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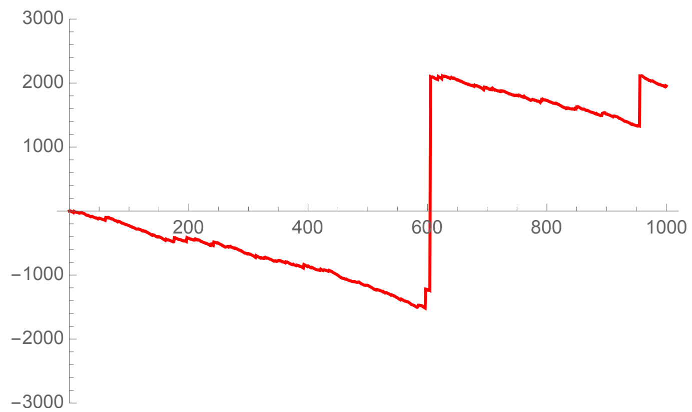

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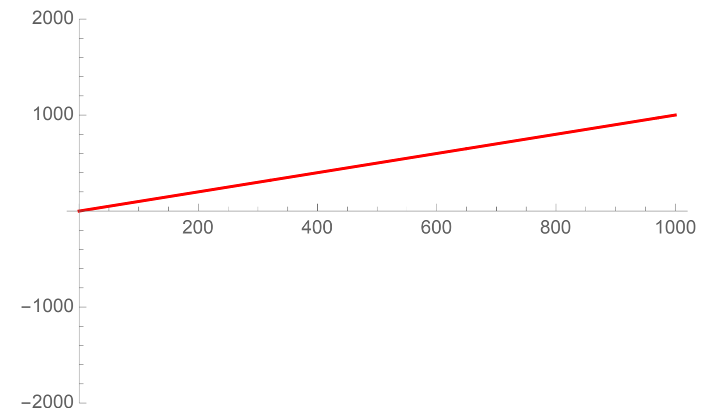

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Probability theory is more concerned with the realm of the probable, rather than the realm of the possible.

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"What happens when $n \to \infty$?"

So, I told him:

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Let E be a Polish space and denote by $\mathcal{M}_1(E)$ the space of all probability measures on E equipped with its Borel σ-field.

Consider the weak-$\star$ topology $\mathcal{M}_1(E)$ for which the convergence $\mu_n \to \mu$ is given by

$$\forall f \in \mathcal{C}_b(E), \qquad \mu_n(f) \to \mu(f).$$

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This course is designed for an audience with a variety of interests:

⇝ for students wishing to deepen their mathematical knowledge of probability theory;

⇝ for students who intend to use it in business applications (a good understanding of probability theory is essential in order to be able to orient oneself in the world of applications and to innovate there).

Why ?

I. A dream

II. Law of large numbers

The law of large numbers

Imagine that you are given a coin. How can you find out if it is rigged or not?

The law of large numbers

Imagine that you are given a coin. How can you find out if it is rigged or not?

Throw n times in a row the coin.

The law of large numbers

A bit more formally, for $i \geqslant 1$ set $X_i = 1$ if the i-th throw is heads (happens with a certain probability p) and 0 if it is tails (happens with probability $1 - p$). Set $S_n = X_1 + \cdots + X_n$.

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$\leadsto$ What is the behavior of $\frac{S_n}{n}$ as $n \to \infty$?

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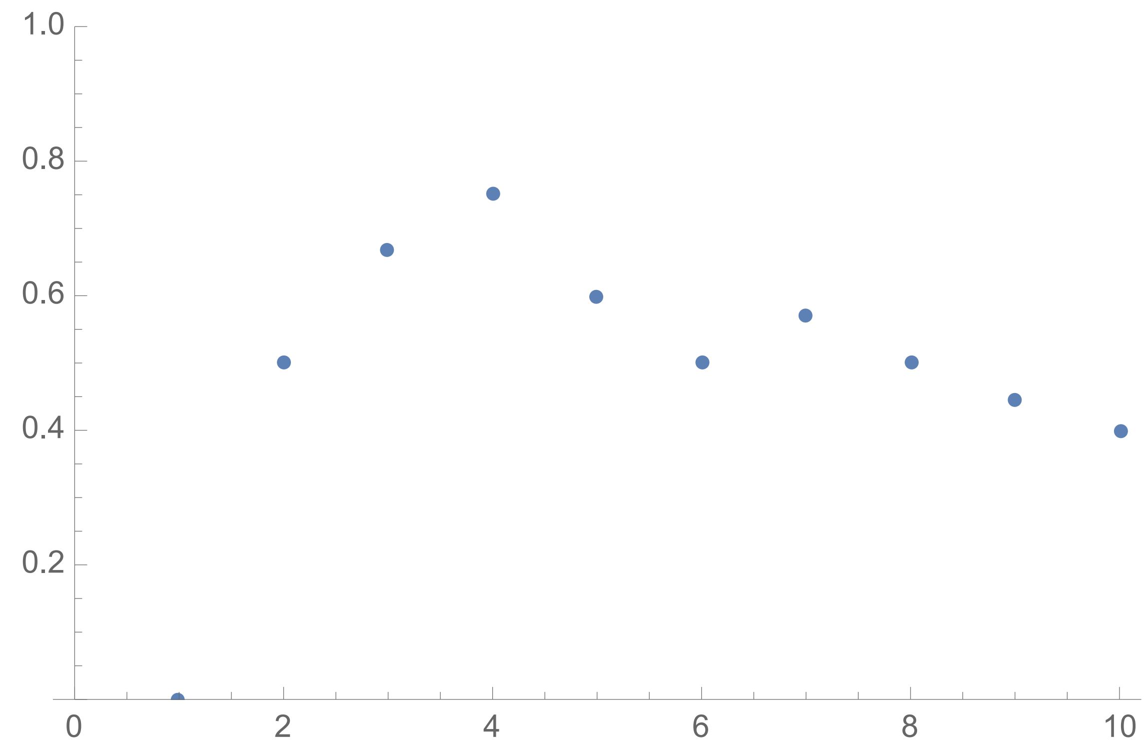

Figure: Simulation of $\left(\frac{S_n}{n} : 1 \leqslant n \leqslant 10\right)$.

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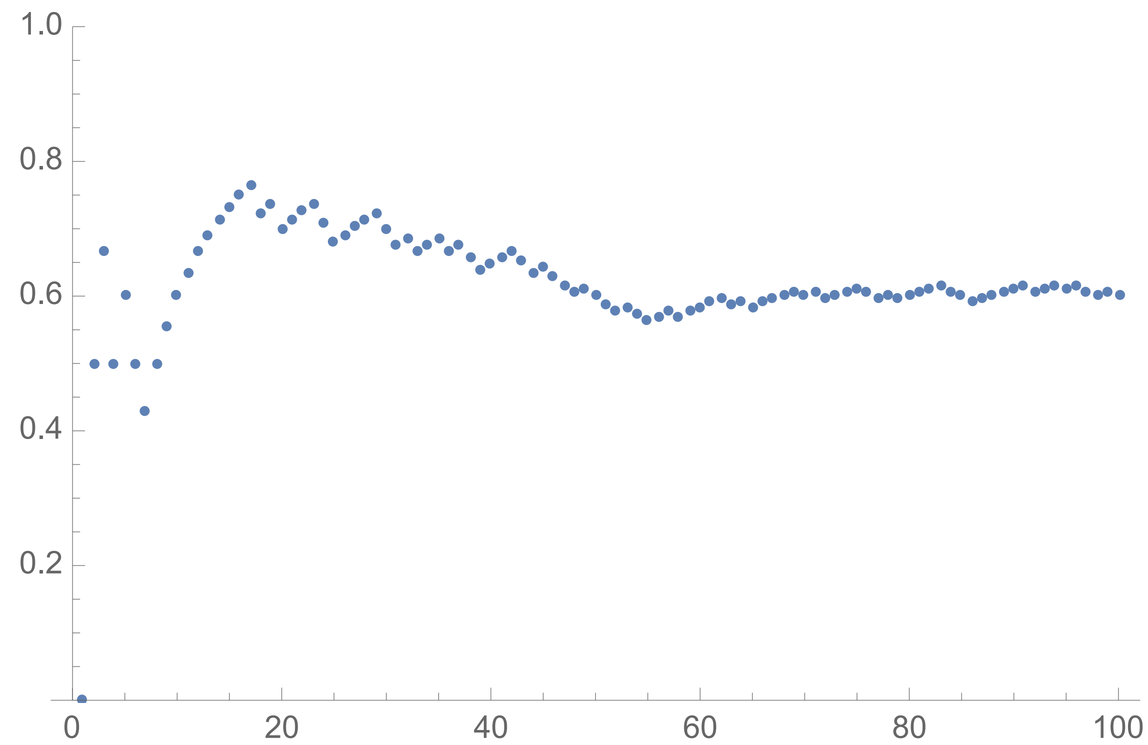

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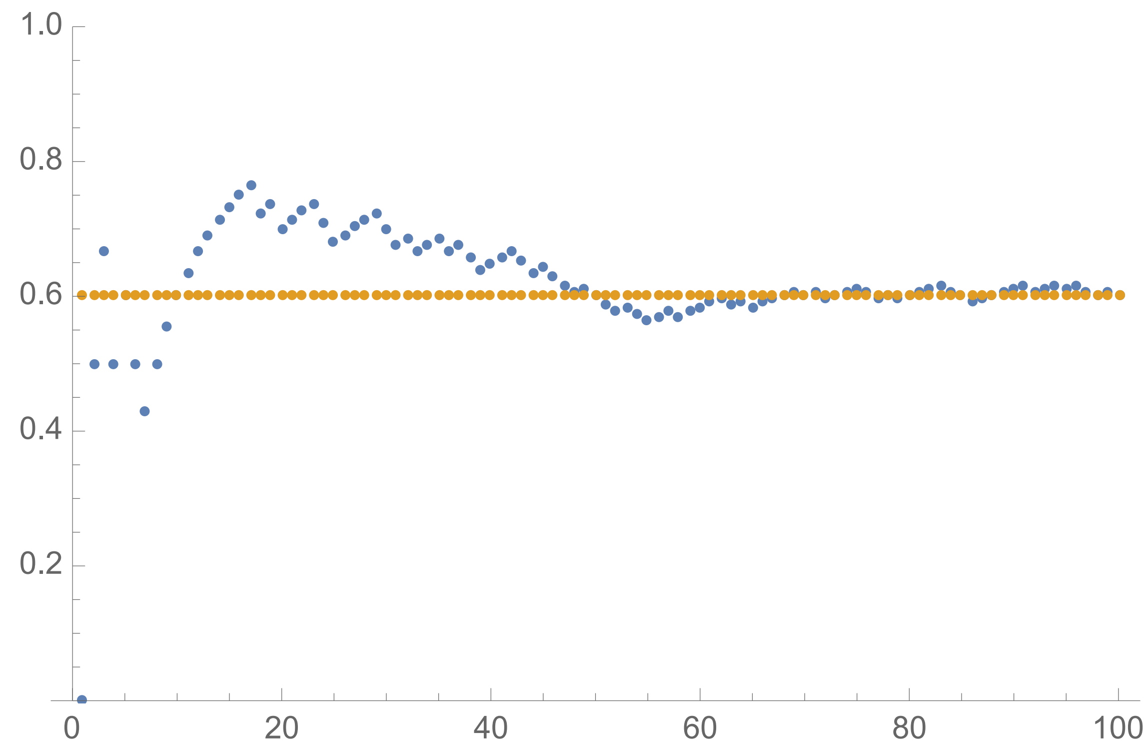

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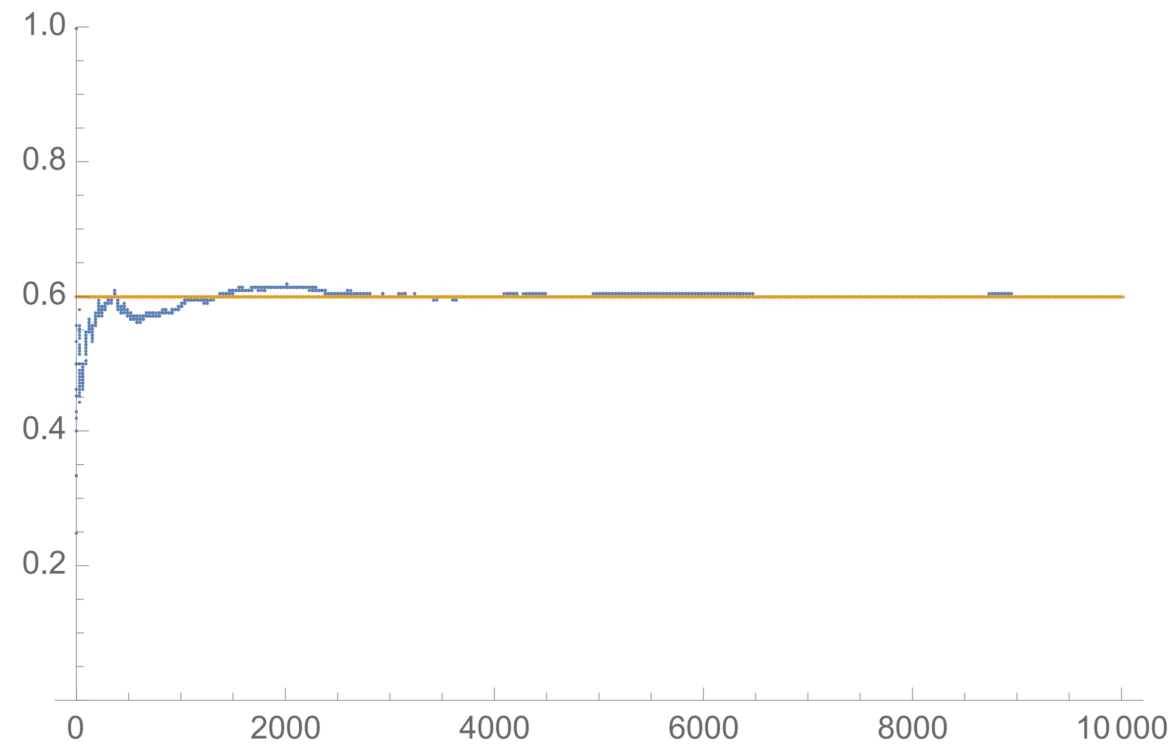

Figure: Simulation of $\left(\frac{S_n}{n} : 1 \leqslant n \leqslant 10000\right)$

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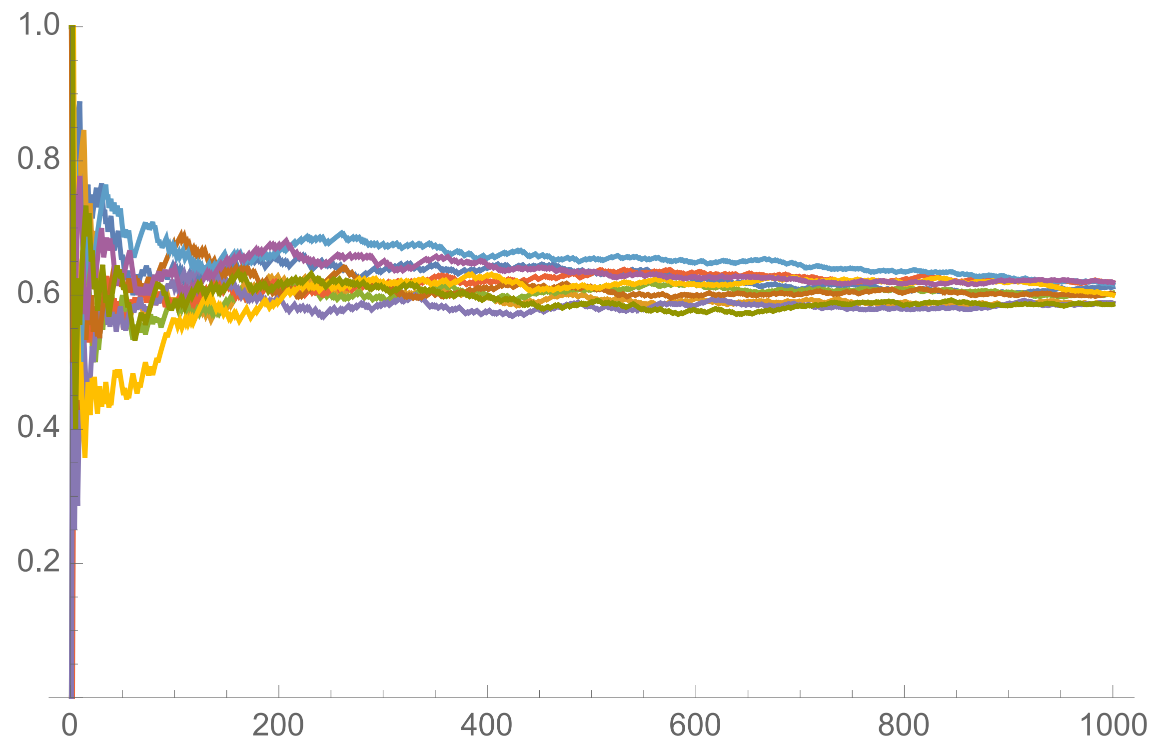

Figure: 10 simulations of $\left(\frac{S_n}{n} : 1 \leqslant n \leqslant 1000\right)$.

I. A dream

II. Law of large numbers

III. Central limit theorem

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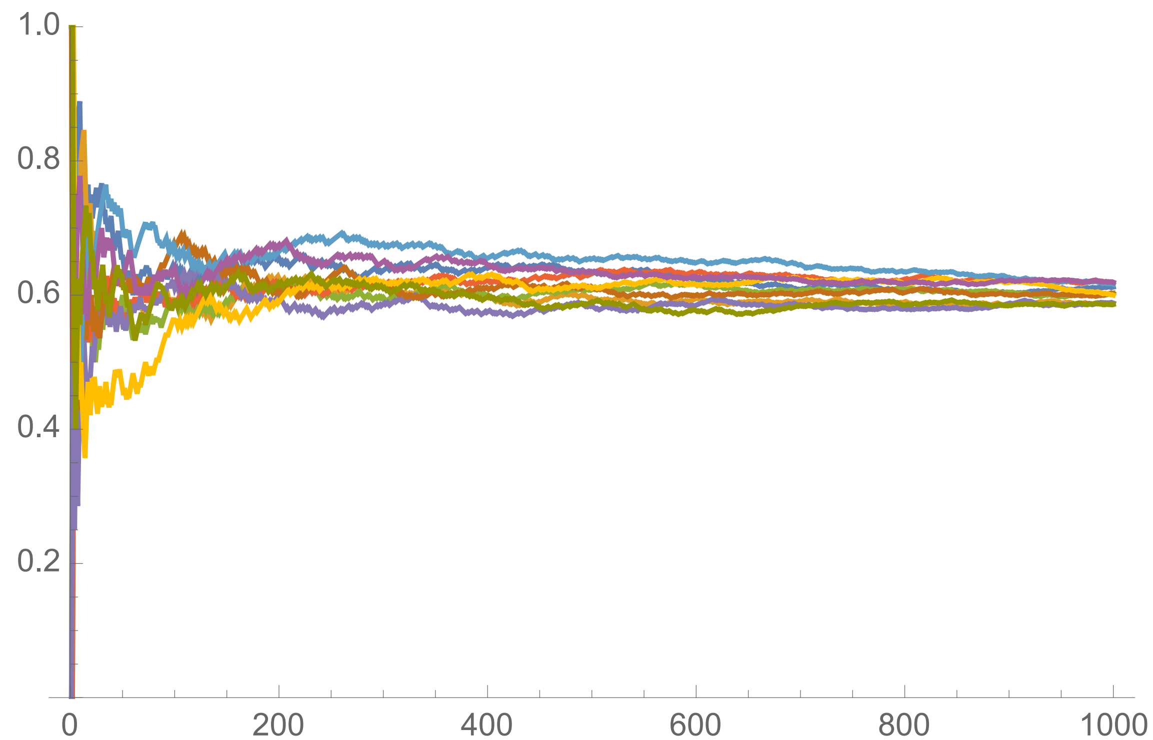

Figure: 10 simulations of $\left(\frac{S_n}{n} : 1 \leqslant n \leqslant 1000\right)$.

↝ Law of large numbers: $\frac{S_n}{n}$ converges **almost surely** towards p as $n \to \infty$.

The Central Limit Theorem

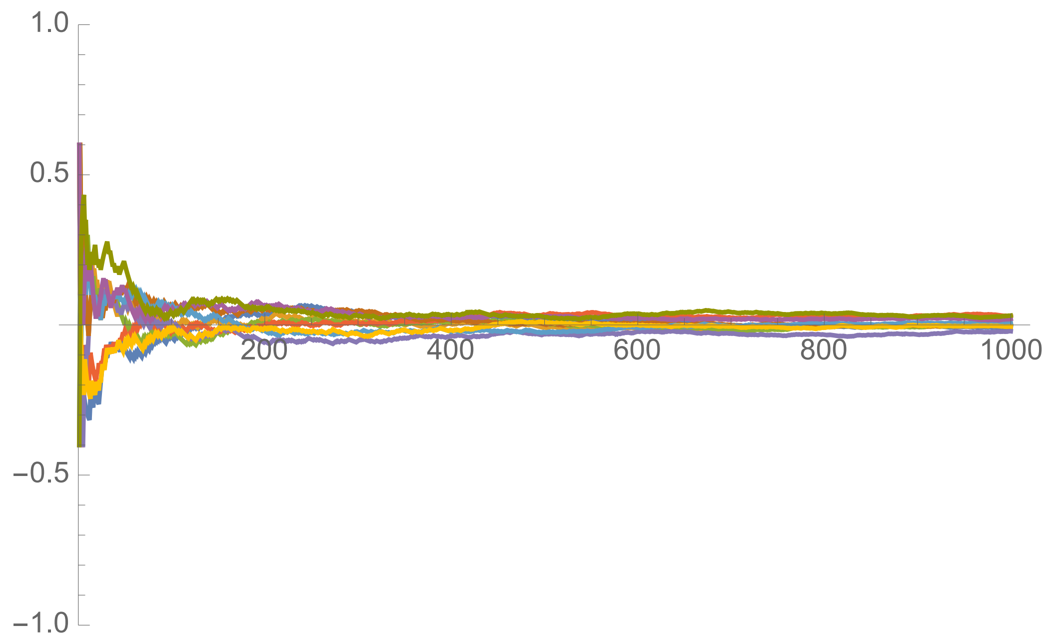

Figure: 10 simulations of $\left(\frac{S_n}{n} - p : 1 \leqslant n \leqslant 1000\right)$ for $p = 0.6$.

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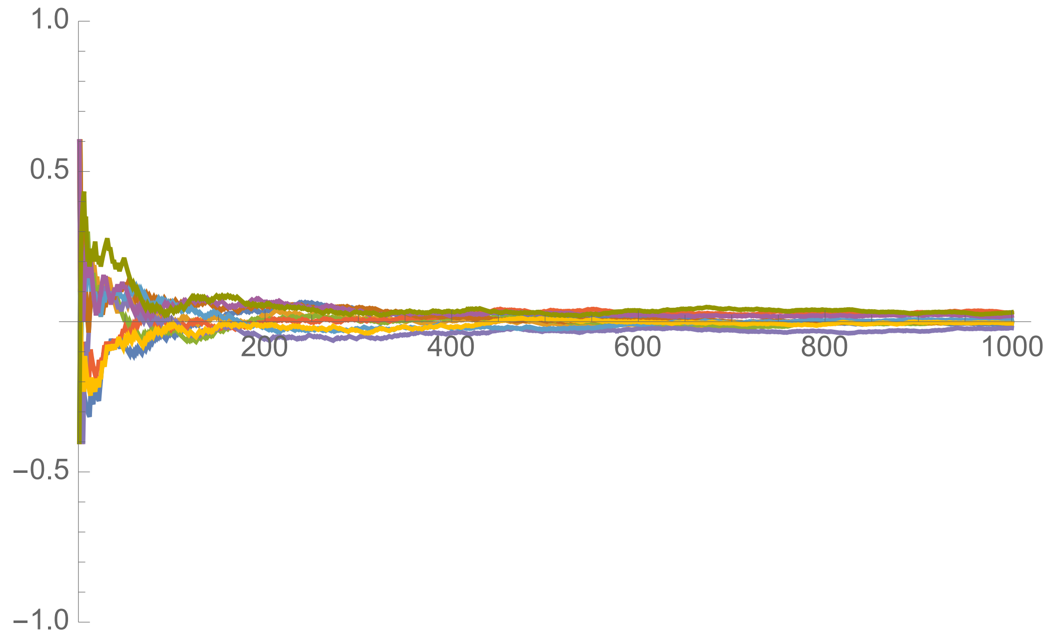

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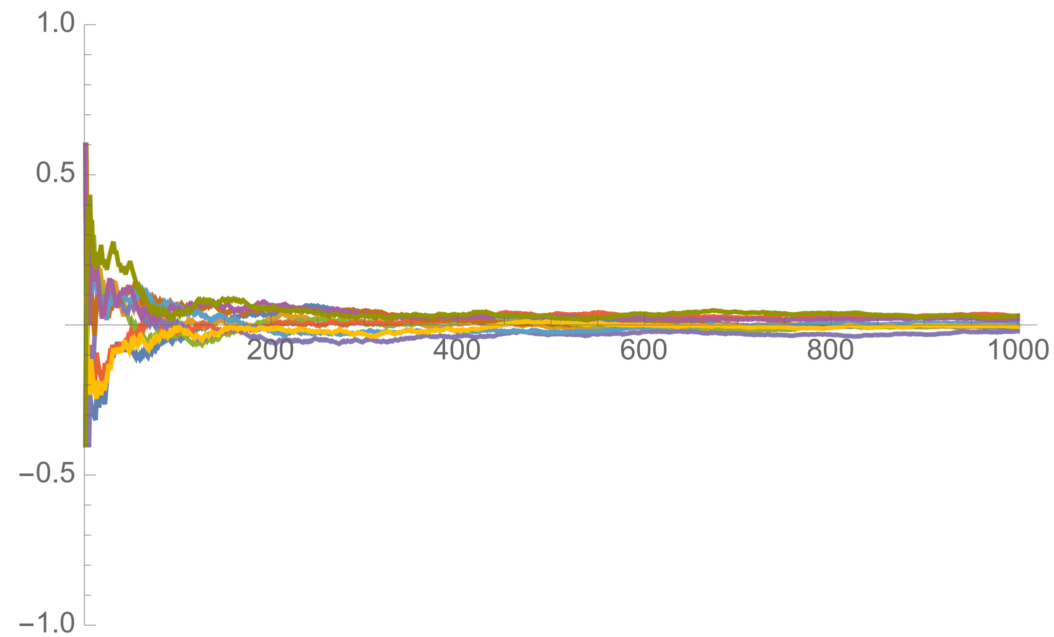
The Central Limit Theorem



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⇝ Is there a function $f(n)$ such that $f(n)\left(\frac{S_n}{n} - p\right)$ has a nice behavior for n large?

The Central Limit Theorem

The speed of convergence is $\frac{1}{\sqrt{n}}$. This means that $\sqrt{n}(\frac{S_n}{n} - p)$ has a non-degenerate behavior as $n \to \infty$.

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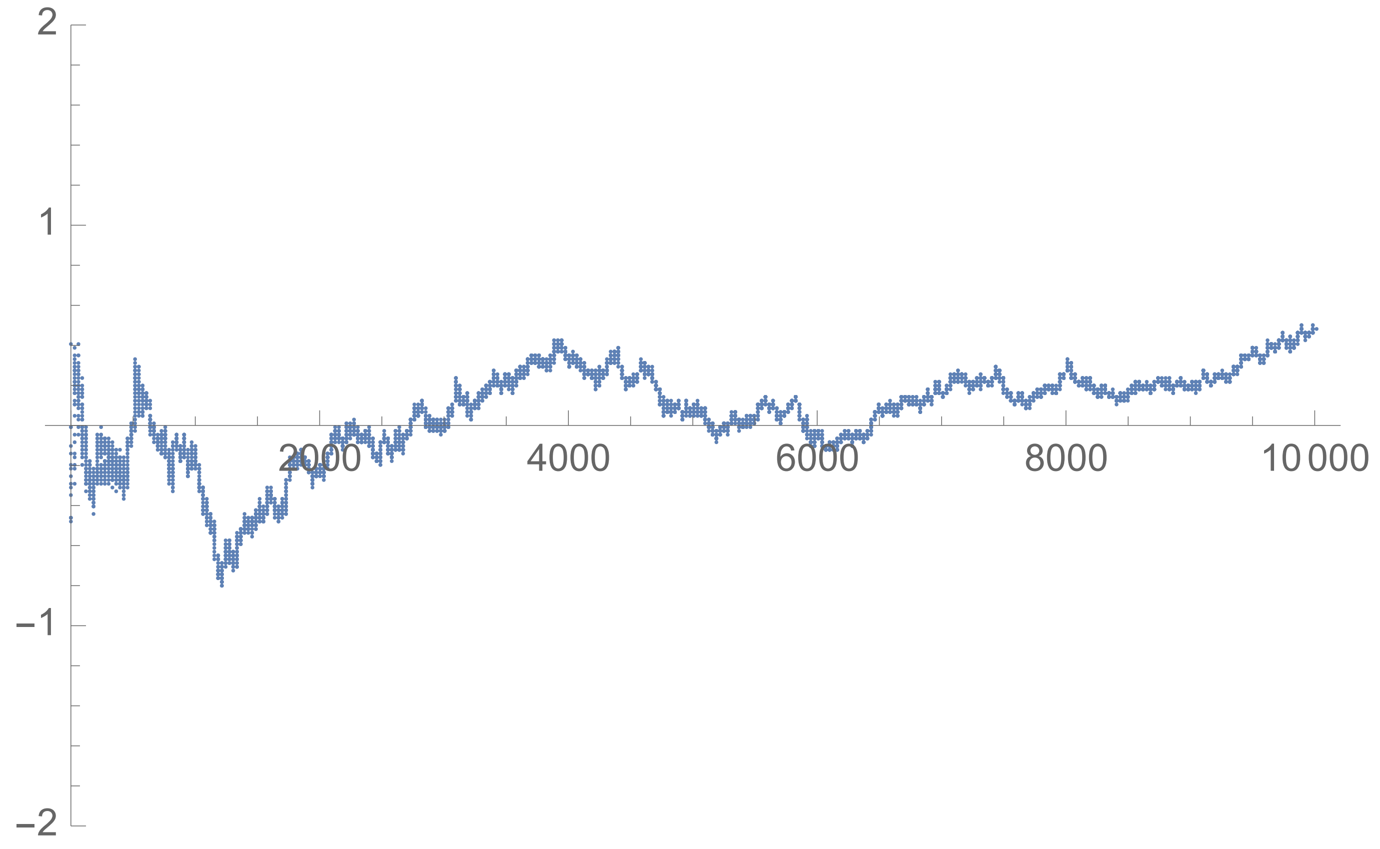

Figure: Simulation of $\left(\sqrt{n} \cdot \left(\frac{S_n}{n} - p\right) : 1 \leqslant n \leqslant 10000\right)$ for $p = 0.6$.

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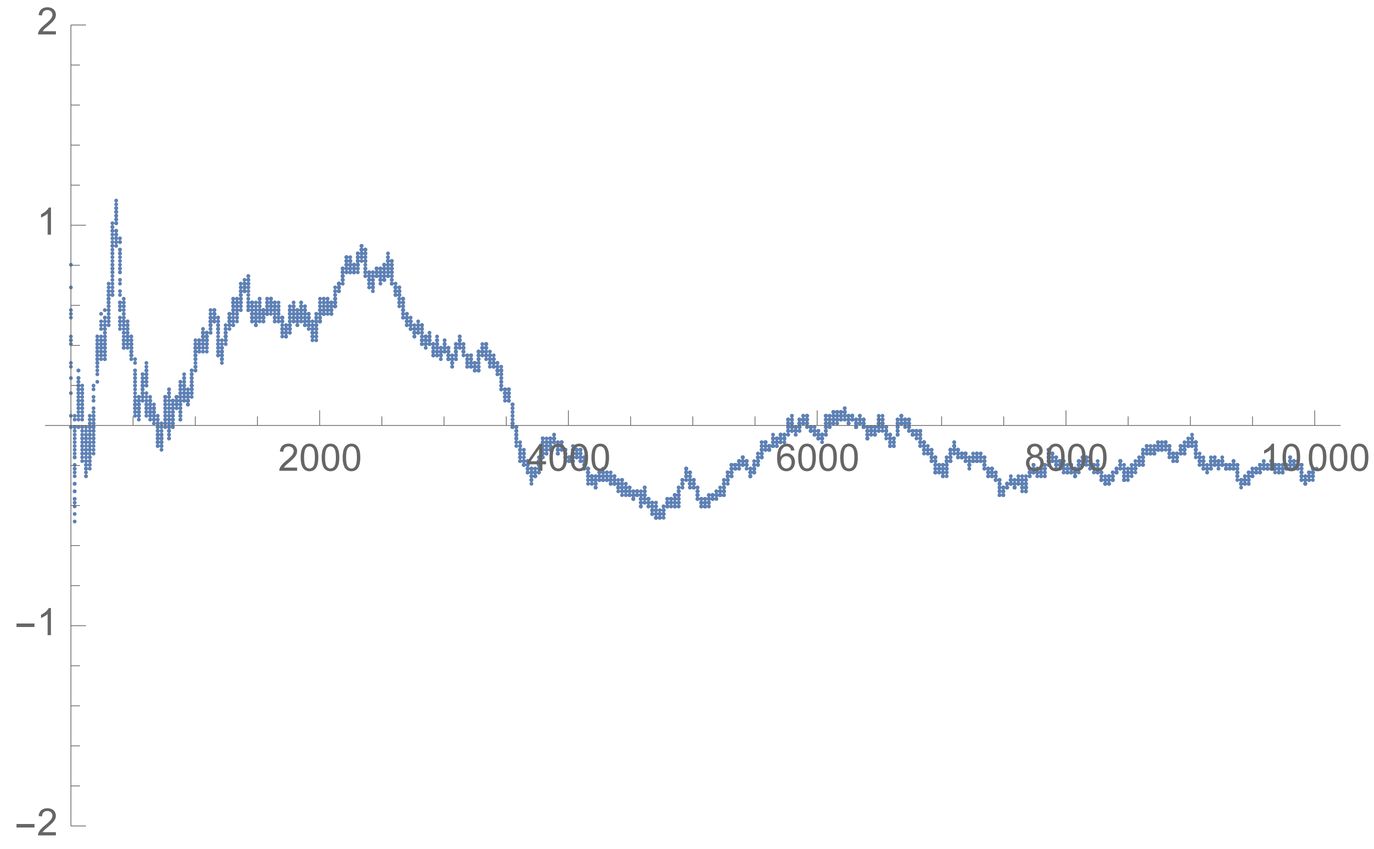

Figure: Another simulation of $\left(\sqrt{n} \cdot \left(\frac{S_n}{n} - p\right) : 1 \leqslant n \leqslant 10000\right)$ for $p = 0.6$.

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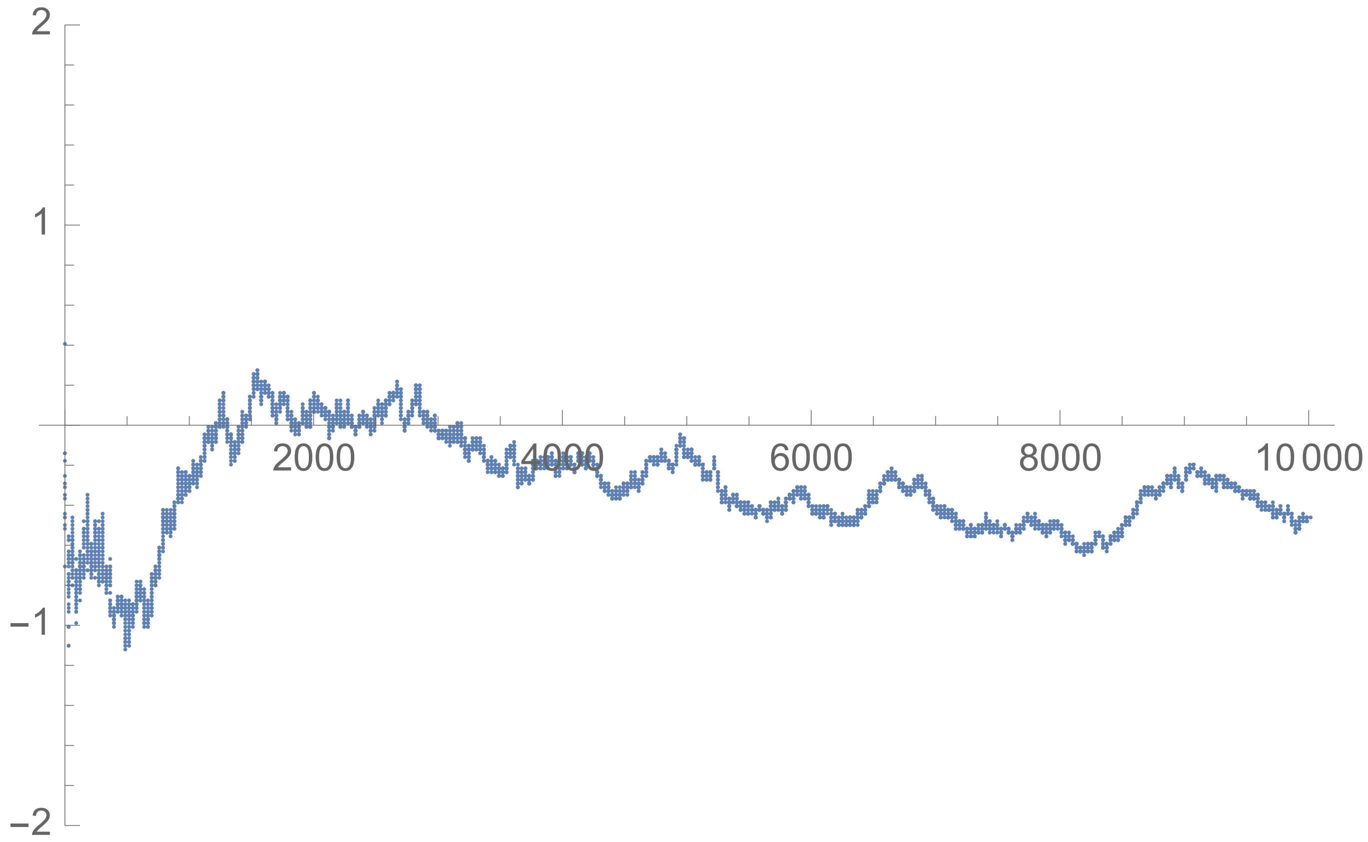

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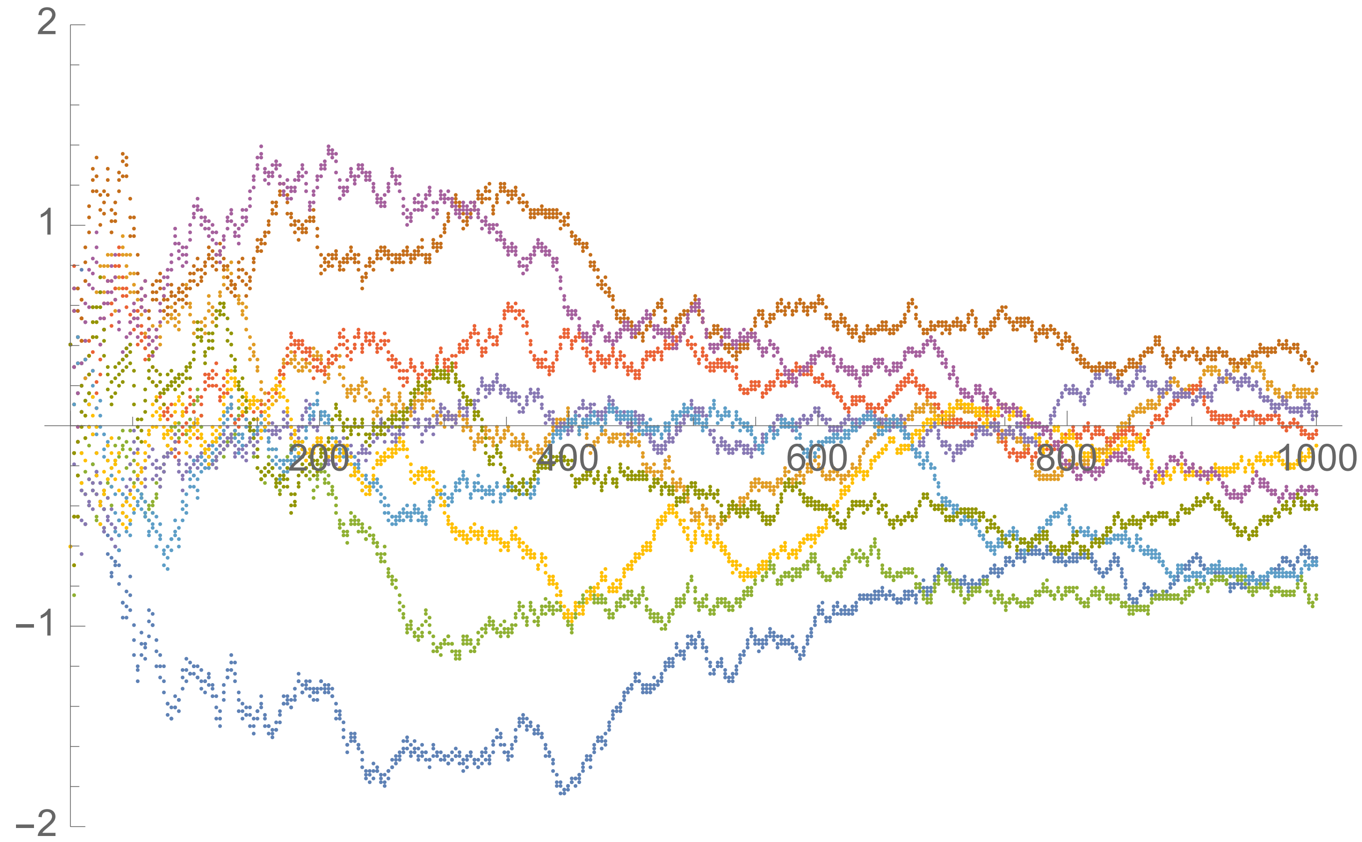

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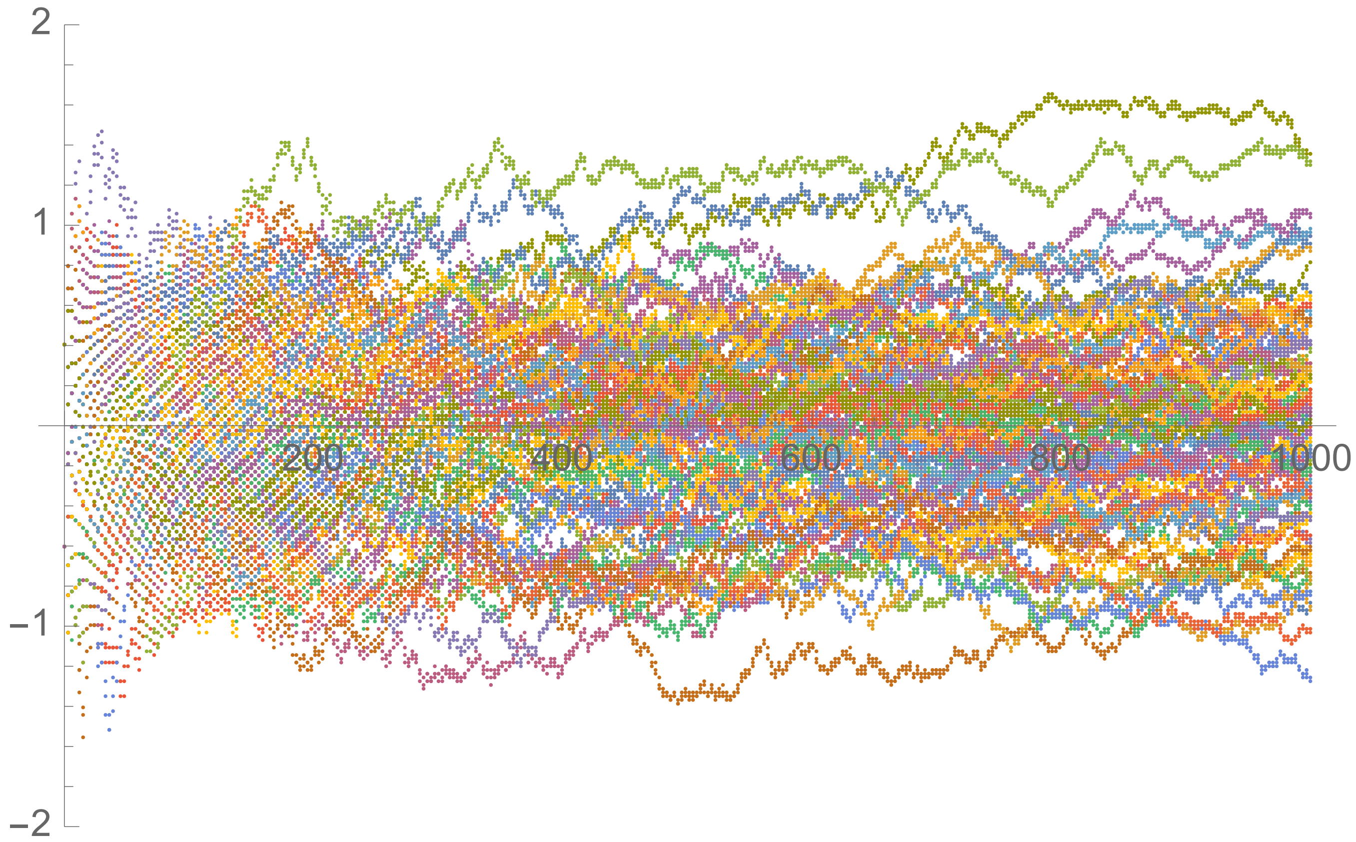
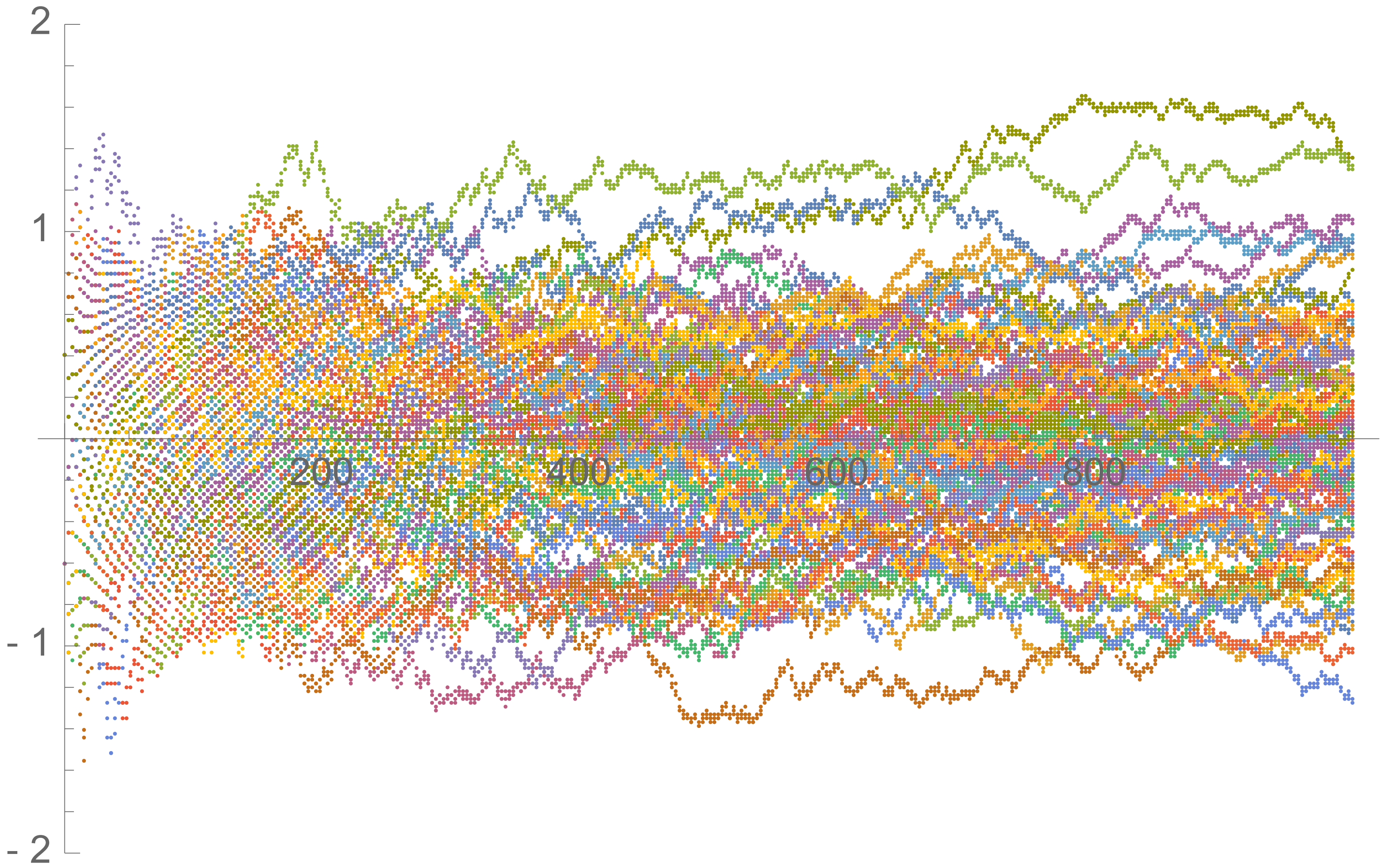


Figure: 100 simulations of $\left(\sqrt{n} \cdot \left(\frac{S_n}{n} - p\right) : 1 \leqslant n \leqslant 1000\right)$ for $p = 0.6$.

Structure in randomness

There is structure in randomness!



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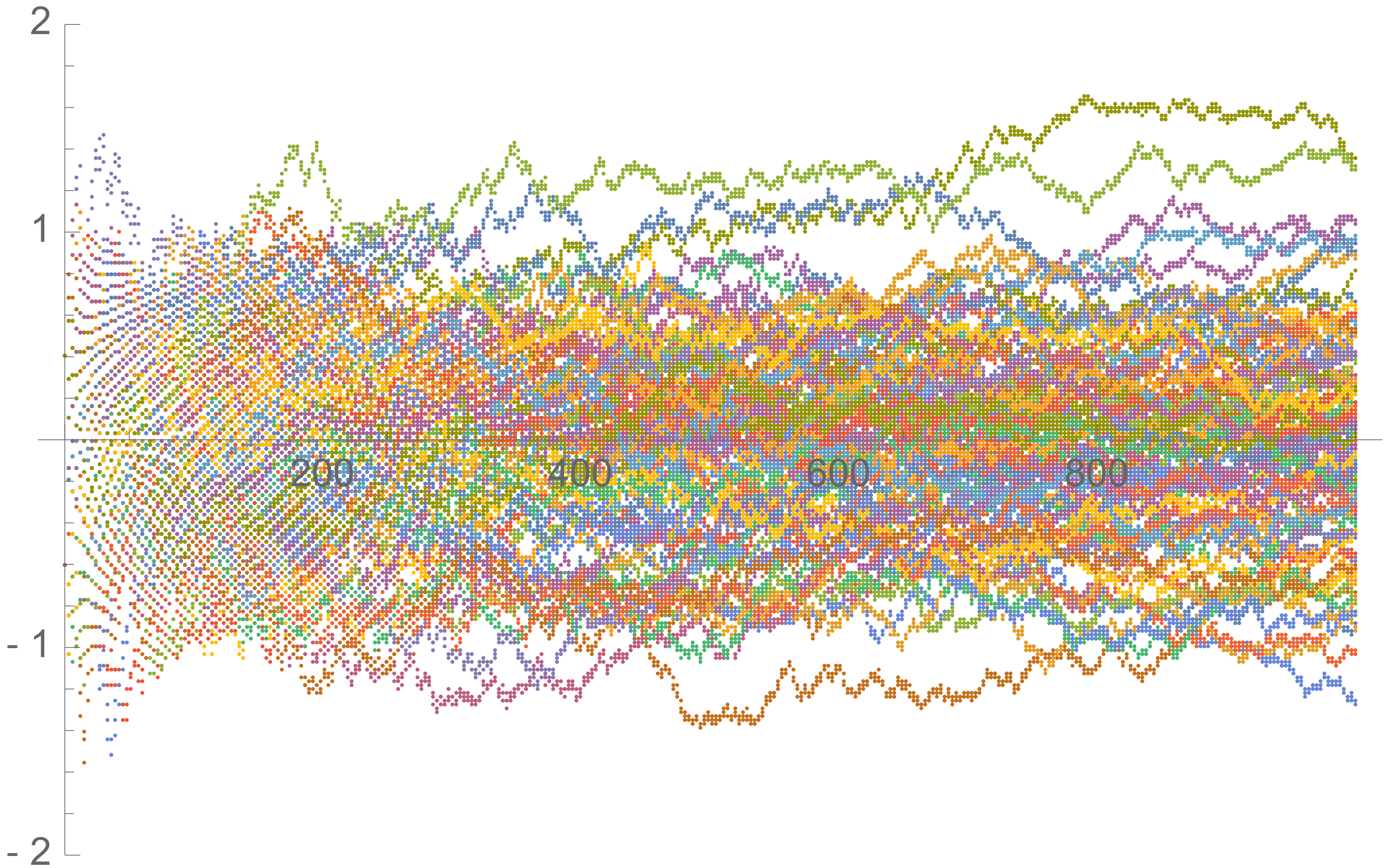

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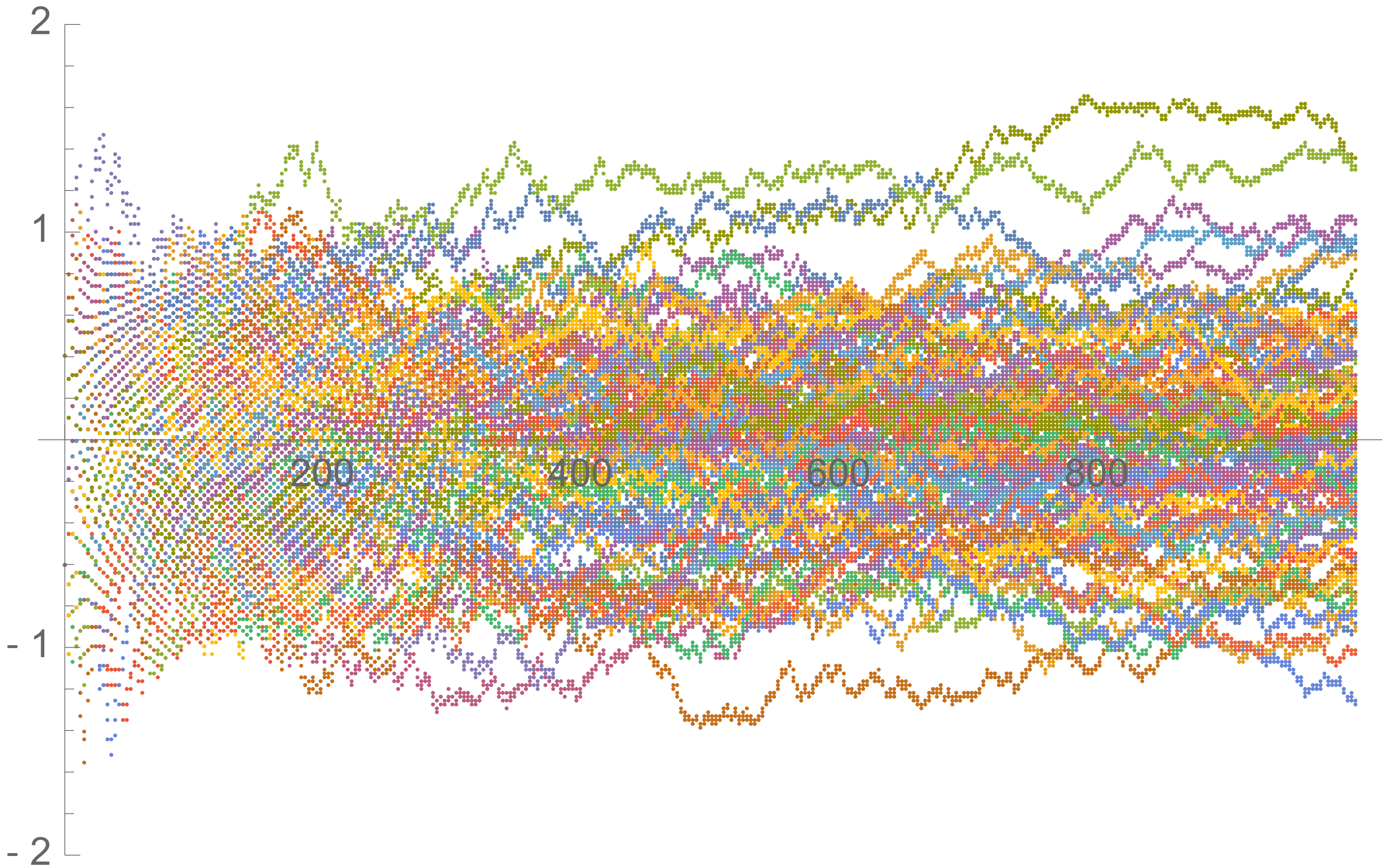

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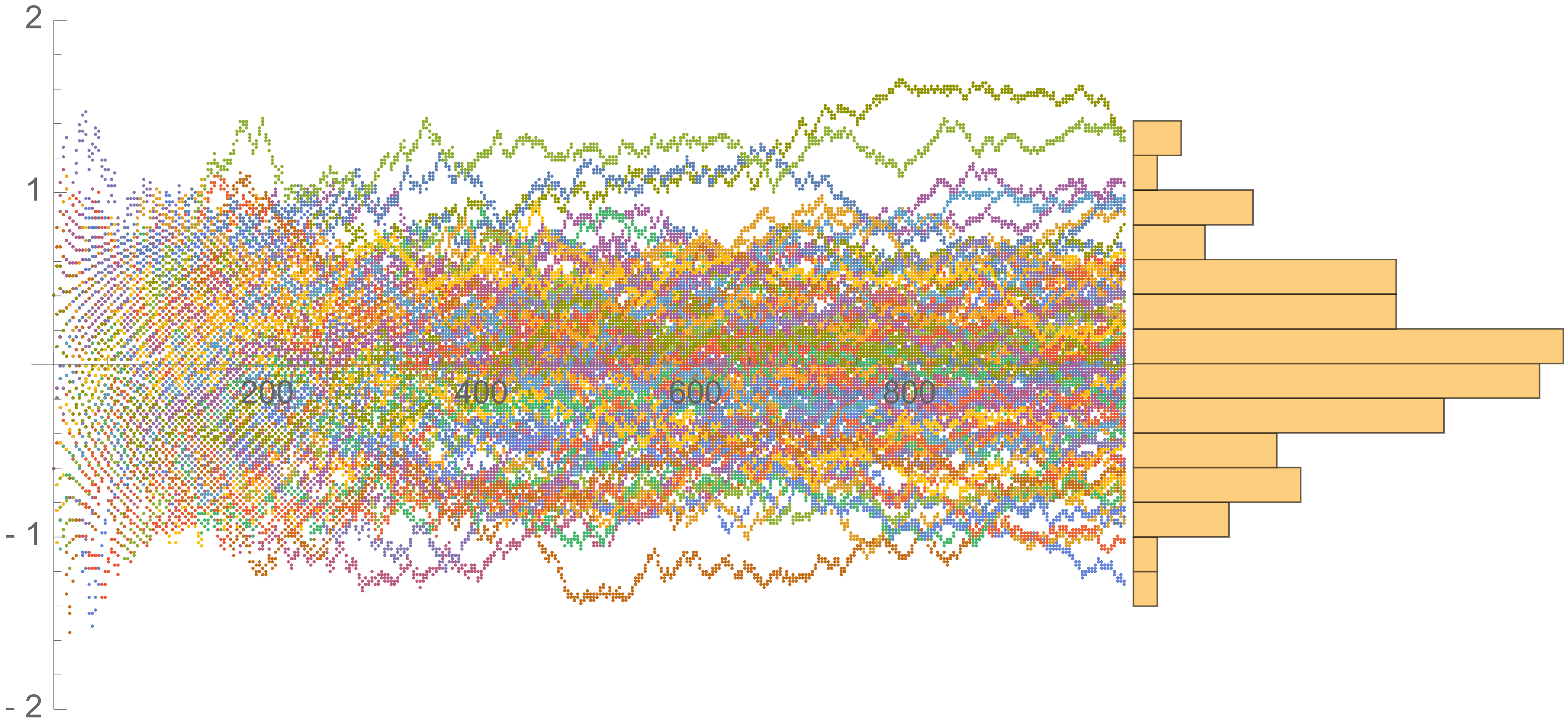

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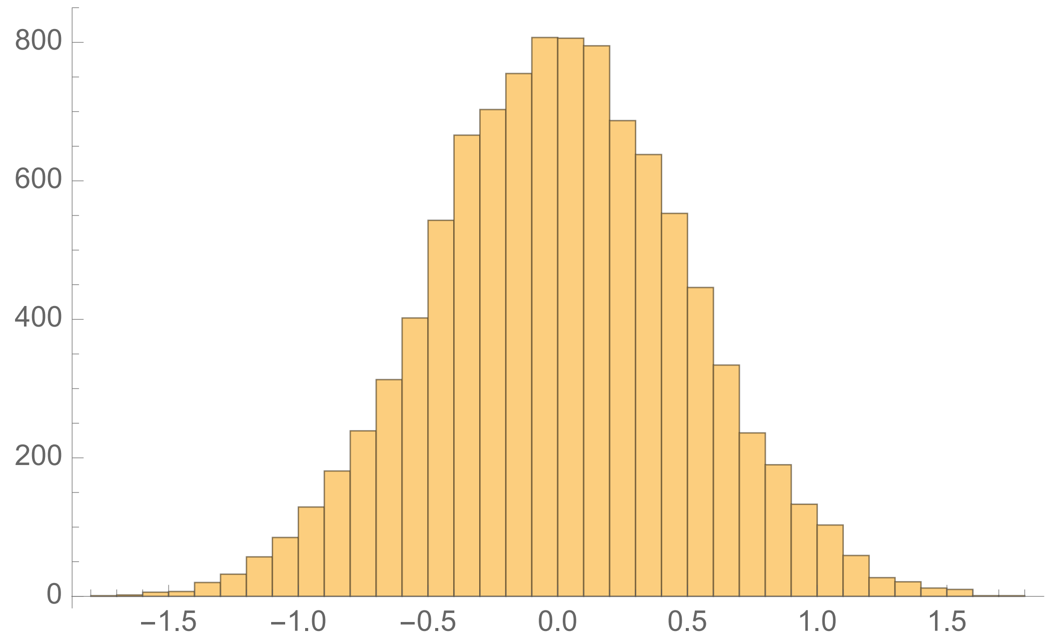


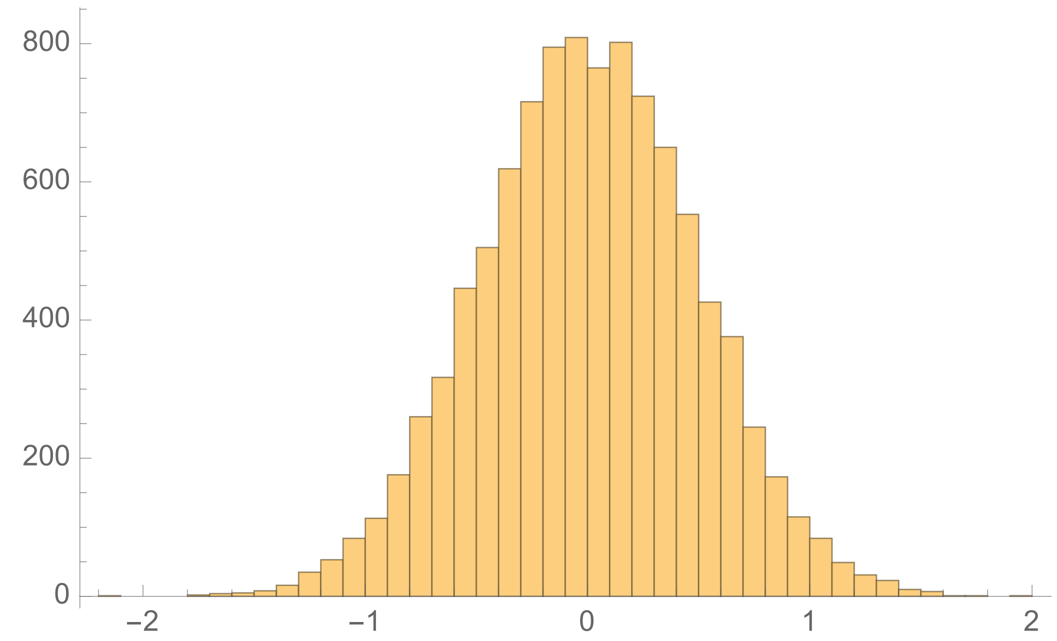
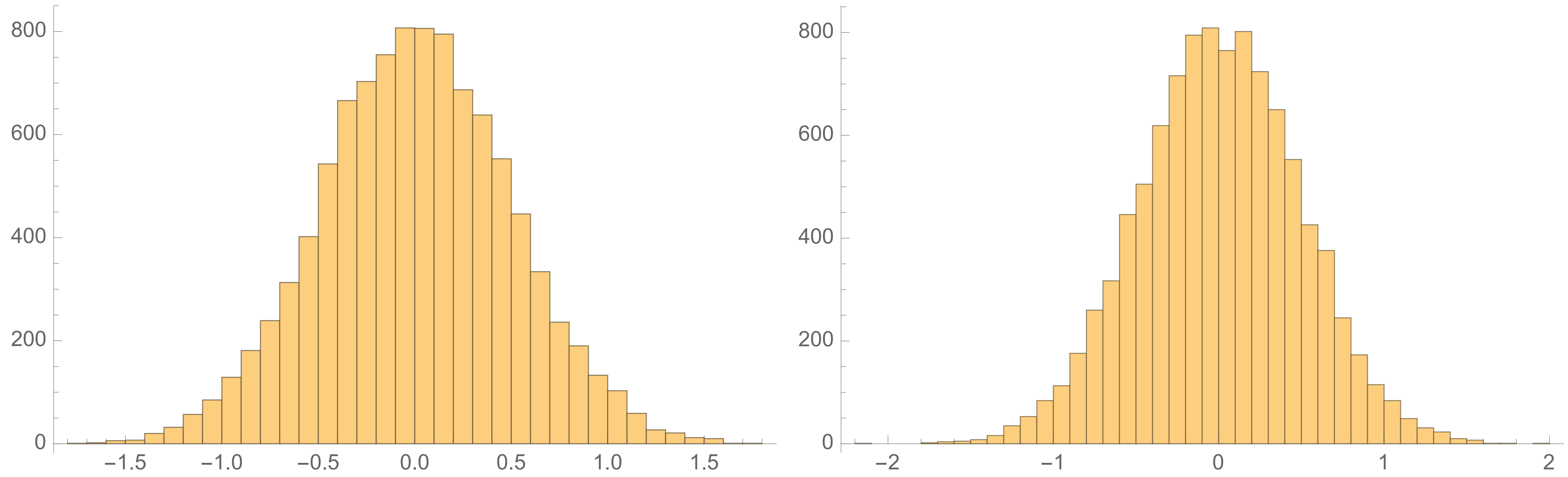


Figure: Empirical histograms of 10000 simulations of $\sqrt{n} \cdot \left(\frac{S_n}{n} - p\right)$ for $n = 10000$. Left: $p = 0.6$; Right: $p = 0.4$.

Structure in randomness



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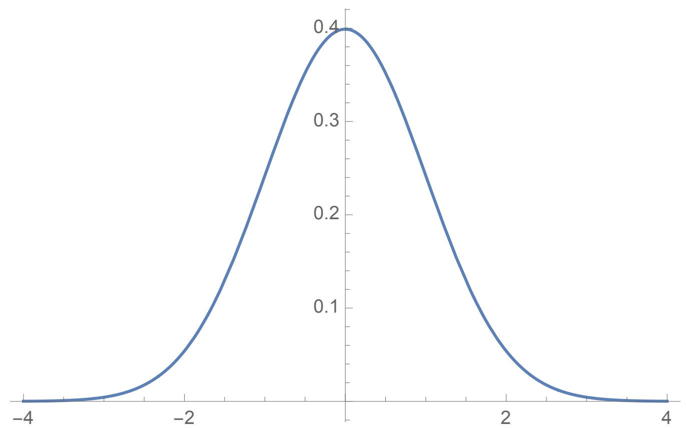
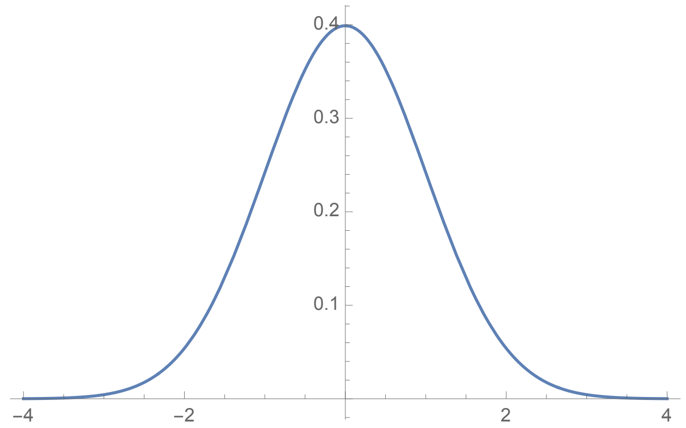


Figure: Plot of the function $x \mapsto \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$.



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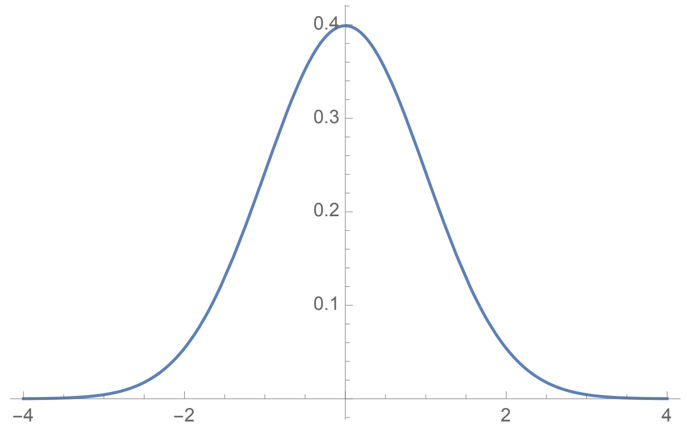

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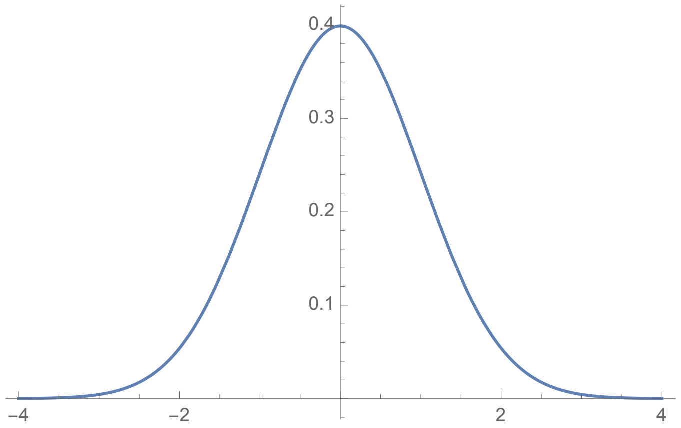
Théorème (Central limit theorem – De Moivre Laplace theorem)**.**

Let S_n be the sum of n independent Bernoulli random variables of parameter $p \in (0,1)$. Then, for every $a < b$:

$$\mathbb{P}\left(a \leqslant \frac{\sqrt{n}}{\sqrt{p(1-p)}}\left(\frac{S_n}{n} - p\right) \leqslant b\right) \quad \underset{n\to\infty}{\longrightarrow} \quad \int_a^b \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx.$$



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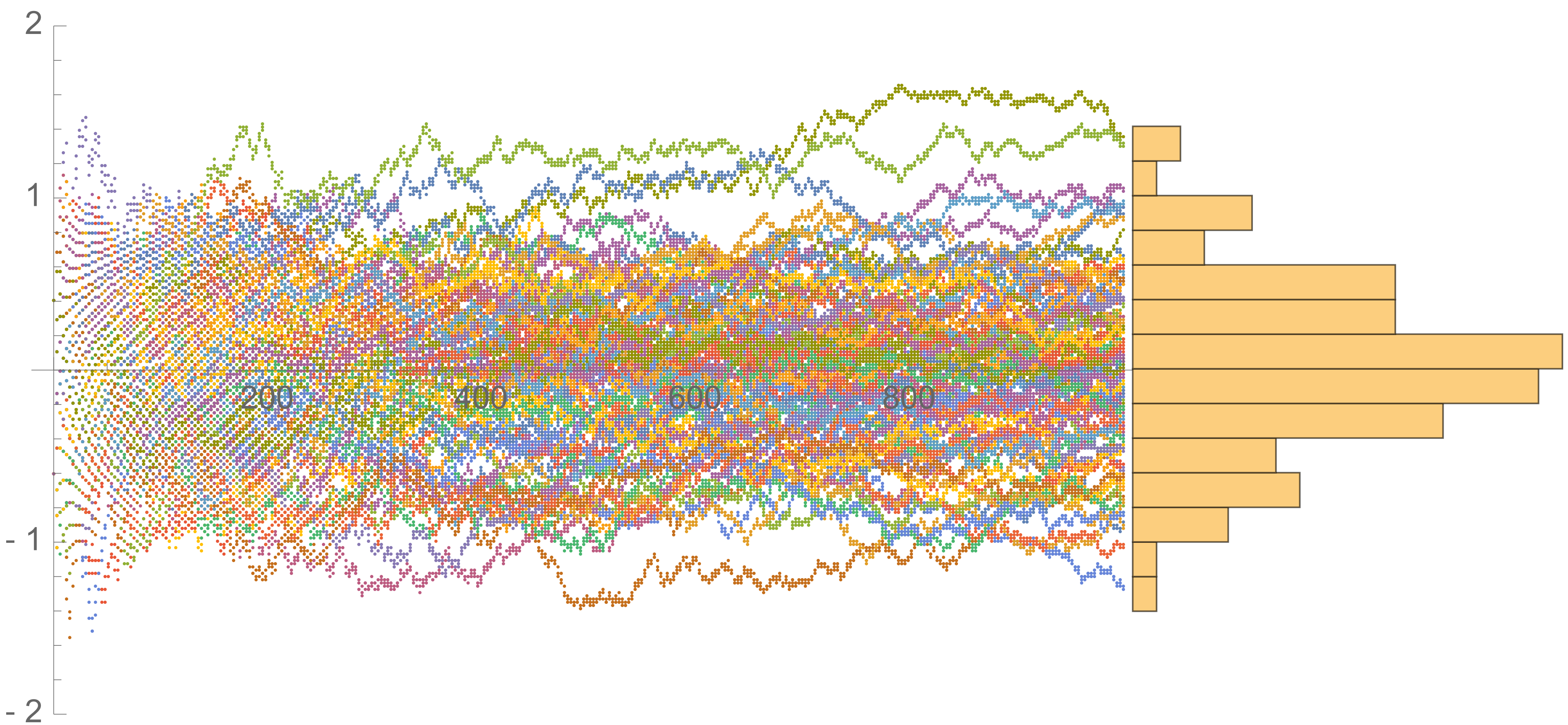
We say that $\frac{\sqrt{n}}{\sqrt{p(1-p)}}\left(\frac{S_n}{n} - p\right)$ converges *in distribution* to a Gaussian random variable.

The central limit theorem



We cannot predict where the endpoint will be, but we know the probability that it belongs to a given region.

I. A dream

II. Law of large numbers

III. Central limit theorem

IV. Practical organization

Probability Theory Autumn 2023 – practical organization

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Lecturer:

⇝ Igor Kortchemski, igor.kortchemski@math.ethz.ch

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⇝ wooclap.com ; code **probability** (questions on your background).