

Mathematical Finance

Exercise Sheet 7

Submit by 12:00 on Wednesday, November 15 via the course homepage.

Exercise 7.1 (*Admissibility at expiry*) Let S be a semimartingale satisfying (NA), and suppose $\vartheta \in \Theta_{\text{adm}}$ has $G_T(\vartheta) \geq -a$ P -a.s. for some $a \geq 0$. Show that $G(\vartheta) \geq -a$ P -a.s., i.e. that ϑ is a -admissible.

Exercise 7.2 (*All gains are zero*)

- (a) Construct an example where S is a martingale, but $\mathcal{G}_{\text{adm}} = \{0\}$. You may use part (b).
- (b) Show that if any continuous adapted process is deterministic, then so is any predictable process.

Exercise 7.3 (*From σ -martingale to local martingale*) Argue in detail that every continuous σ -martingale null at zero is a local martingale null at zero.

Can you find an example where it is not a supermartingale?

Exercise 7.4 (*Theorem 4.5*) Let S be a semimartingale. Prove (3) \implies (1) in Theorem 4.5, i.e. the existence of an equivalent σ -martingale measure for S implies (NFLVR).