Problems marked with a (*) are a bit harder. Hints are available in the next page.

1.1. Recap Questions.

1. Let $\{x_k\}_{k\in\mathbb{N}} \subset [1,2] \subset \mathbb{R}$ be a sequence. Is it necessarily convergent?
2. Describe all differentiable functions $f\colon \mathbb{R}\setminus\{0\} \to \mathbb{R}$ such that $f'(x) = 1/x$ for all $x \neq 0$.
3. Let $f\colon \mathbb{R} \to \mathbb{R}$ be continuous and bounded, is it true that
$$\int_0^{\pi/2} f(\cos(x))\,dx = \int_0^1 \frac{f(t)}{\sqrt{1-t^2}}\,dt?$$
4. Let $f\colon [0,\infty) \to \mathbb{R}$ have $f''(x) > 0$ for all $x \geq 0$. Can f have a global maximum point?
5. Let $\phi\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map and let $Q := [0,1]\times[0,1]$ be the unit square. How does $\phi(Q)$ look like? What are the possibilities?
6. Consider the vector space V of polynomials of degree less than 4, and the function $D\colon V \to V$ which associates to a polynomial p its derivative p'. Is D a linear map? Is so, what is the dimension of its null space?

1.2. From equations to drawings. Sketch the following subsets of $\mathbb{R}^2, \mathbb{R}^3$.

1. $A := \{(x,y) : x^2+y^2 < 1,\ y \geq x^2\}$
2. $B := \{(x,y) : y^2 = x^3 - x\}$
3. $C := \{(x,y) : \frac{\pi}{6} < \arctan(y/x) < \frac{\pi}{3},\ 0 < x < y\}$
4. $D := \{(x,y,z) : \sqrt{x^2+y^2} = 1 - z, 0 < z < 1\}$
5. $E := \{(x,y,z) : z = \sqrt{1+x^2+y^2}\}$
6. $F := \{(x,y,z) : z = \max\{|x|,|y|\}\}$

1.3. From drawings to equations. Express in terms of functions and equations the following sets.

1. The coordinates of a point in the plane that goes endlessly back and forth from the origin to the point $(-1,1)$.
2. A planar domain with two holes in it.
3. A bi-dimensional disk sitting inside the plane $\{x+y+z=0\}$ in $\mathbb{R}^3$.
4. A Pac-Man shaped domain in the plane.
5. A pyramid, whose base is a square, sitting in $\mathbb{R}^3$.

Hints:

1.2 As ETH students you have a license for Mathematica. All these plots are one-line-commands using that software!

1.2.2 The set is clearly symmetric with respect to $y \mapsto -y$, so it's enough to plot qualitatively the function $y(x) = \sqrt{x^3 - x}$, separately in the intervals $x \in [-1, 0]$ and $x \in [0, \infty)$... From here you can use all the technology from Analysis I...

1.2.4 Try to figure out how D looks layerwise, i.e., as a planar domain once z is kept fixed.

1.2.5 If you known how to plot $z = f(s), s \geq 0$, then the set $\{(x, y, z) : z = f(\sqrt{x^2 + y^2})\}$ is obtained rotating the plot of f around the z-axis...

1.3.1 Construct an appropriate function $f : \mathbb{R} \to \mathbb{R}^2$, where t represents the time and $f(t)$ represents the position of the moving point at time t. You want f to be periodic in t and $f(t)$ to be aligned with the direction $(-1, 0)$

1.3.3 Observation: the intersection of a round ball with a plane is a disk...

1.3.5 Take inspiration from 1.2.4 and 1.2.6...