Single Choice 1

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1. Which of the following statements is true?

(a) In any ring R, the zero ideal (0) is a prime ideal.
(b) Each principal ideal domain is an Euclidean domain.
(c) For all fields K and M, each ring homomorphism $K \to M$ is injective.
(d) For all fields K and M, each ring homomorphism $K \to M$ is surjective.

2. Which subring of $\mathbb{C}$ is **not** equal to the others?

(a) $\mathbb{Z}[\frac{i}{20}, -10i]$
(b) $\mathbb{Z}[\frac{1}{2}, \frac{i}{5}]$
(c) $\mathbb{Z}[\frac{1}{2}, 10i]$
(d) $\mathbb{Z}[\frac{i}{20}, \frac{10}{i}]$

3. $\mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}$ is a group under componentwise addition. Consider the subgroup

$$H := \{h \cdot (1, 2, 3) \mid h \in \mathbb{Q}\}.$$

Then $(\mathbb{Q} \times \mathbb{Q} \times \mathbb{Q})/H$ is isomorphic to

(a) $\mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}$
(b) $\mathbb{Q} \times \mathbb{Q}$
(c) $\mathbb{Q}$
(d) $\{0\}$

4. Which of the following ideals in $\mathbb{Q}[X]$ is not a maximal ideal?

(a) $(X + 1)$
(b) $(X^2 + 1)$
(c) $(X^3 + 1)$
(d) $(X^4 + 1)$

5. Let R^* be the group of units. Which of the following is true for each $x \in R^*$?

(a) $x + 1 \in R^*$
(b) $x^2 \in R^*$
(c) $\forall y \in R\ \exists z \in R : y \cdot z = x$
(d) All the statements above are true.