Single Choice 10

1. The group $\mathrm{Gal}(\mathbb{Q}(\sqrt[4]{2}, i) : \mathbb{Q})$ is isomorphic to...

(a) $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$

(b) $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$

(c) $\mathbb{Z}/8\mathbb{Z}$

(d) D_4

2. Let $L : K$ be a Galois extension with Galois group G. Let $a \in L$ be given. Which of the following statements is false?

(a) Let $\sigma \in G$ be a non-trivial element. If G is cyclic and $\sigma(a) = a$, then $a \in K$.

(b) The element
$$\sum_{\varphi \in G} \varphi(a)$$
is in K.

(c) If the set $\{\varphi(a) \mid \varphi \in G\}$ contains at most two elements, then $[K(a) : K] \leqslant 2$.

(d) If $\varphi(a) = a$ for all $\varphi \in G$, then $a \in K$.

3. Which of the following extensions is a normal closure of $\mathbb{Q}(\sqrt[4]{5}) : \mathbb{Q}$?

(a) $\mathbb{Q}(\sqrt[8]{5}, i\sqrt[4]{5}) : \mathbb{Q}(\sqrt[4]{5})$

(b) $\mathbb{Q}(\sqrt[4]{5}, i\sqrt[2]{5}) : \mathbb{Q}(\sqrt[4]{5})$

(c) $\mathbb{Q}(\sqrt[8]{5}, i) : \mathbb{Q}(\sqrt[4]{5})$

(d) $\mathbb{Q}(\sqrt[4]{5})$ is already a normal closure of $\mathbb{Q}(\sqrt[4]{5}) : \mathbb{Q}$

4. Let $L : K$ be a Galois extension with Galois group G. Which of the following statements is true?

(a) Any subgroup $H \leqslant G$ is the Galois group of some extension $M : K$, for some $M \subset L$.

(b) Any subgroup $H \leqslant G$ is the Galois group of some extension $L : M$, for some $M \subset L$.

(c) For any subgroup $H \leqslant G$ the intermediate extension L^H is a normal extension of K.

(d) None of the statements above is true.

5. Which of the following statements is true for every algebraic field extensions $M : L : K$?

(a) If $M : K$ is Galois, then also $M : L$ is Galois.

(b) If $M : K$ is Galois, then also $L : K$ is Galois.

(c) If $M : L$ and $L : K$ are both Galois, then also $M : K$ is Galois.

(d) All of the statements above are true.