Single Choice 11

1. The Galois group of the splitting field of the polynomial $x^3 + x + 1$ over $\mathbb{Q}$ is isomorphic to...

(a) C_2
(b) C_6
(c) A_3
(d) S_3

2. The order of the Galois group of the splitting field of the polynomial $x^3 - 2x + 1$ over $\mathbb{Q}$ is equal to...

(a) 2
(b) 3
(c) 6
(d) 9

3. Let $f(x) := x^3 - 12x + 34$. Let K be a splitting field of f over $\mathbb{Q}$. Which of the following statements is **false**?

(a) f is irreducible over $\mathbb{Q}$
(b) $\mathrm{Gal}(K : \mathbb{Q})$ contains an element of order 2
(c) f has precisely one real zero
(d) $\mathrm{Gal}(K : \mathbb{Q}) \cong A_3$

4. Let M be a Galois extension of K with $|\mathrm{Gal}(M : K)| = 12$. Which of the following statements is **false**?

(a) There always exists a subfield L of M containing K with $[L : K] = 2$.
(b) There always exists a subfield L of M containing K with $[L : K] = 3$.
(c) There always exists a subfield L of M containing K with $[L : K] = 4$.
(d) There always exists a subfield L of M containing K with $[L : K] = 6$.

5. Which of the following statements is **false**?

(a) Let K be a field, $f \in K[X]$ a polynomial, such that the discriminant of f is non-zero and a perfect square. Then the Galois group of f over K is contained in A_n for $n = \deg(f)$.
(b) If $K : \mathbb{Q}$ is a finite normal extension, then there exists an extension N of K such that every $\mathbb{Q}$-monomorphism $\varphi : K \to N$ is a $\mathbb{Q}$-automorphism of K.
(c) Every finite, separable and normal field extension is Galois.
(d) The 11-th cyclotomic polynomial is equal to $\sum_{k=0}^{10} x^k$.