Single Choice 12

1. Let K be the splitting field of $x^{49} - 1$ over $\mathbb{Q}$. Then $[K : \mathbb{Q}]$ is equal to

(a) 7
(b) 42
(c) 48
(d) 49

2. Let $r \in \mathbb{Z}_{>1}$ and let $L : K$ be a Galois extension such that $\mathrm{Gal}(L : K)$ is cyclic of order 2^r. What is the number of the subfields M such that $K \subsetneq M \subsetneq L$?

(a) $r - 1$
(b) r
(c) $r + 1$
(d) $r + 2$

3. Let K be the splitting field of $x^{42} - 1$ over $\mathbb{Q}$. What is the number of the subfields M such that $\mathbb{Q} \subsetneq M \subsetneq K$?

(a) 3
(b) 4
(c) 6
(d) 8

4. Let $f \in \mathbb{Q}[X]$ be irreducible and let K denote its splitting field. If $\mathrm{Gal}(K : \mathbb{Q}) = D_4$ (the dihedral group of order 8), what are the possibilities for the degree of f?

(a) Only the degree 4 is possible.
(b) Only the degree 8 is possible.
(c) Only the degrees 4 and 8 are possible.
(d) The degrees 2, 4 and 8 are all possible.

5. Which of the following statements is **false**?

(a) There exists a primitive root of unity ζ such that $\mathbb{Q}(\sqrt{5}) \subset \mathbb{Q}(\zeta)$.
(b) Let $K : \mathbb{Q}$ be a finite normal extension. If $\mathrm{Gal}(K : \mathbb{Q})$ is solvable, then there exists an extension $L : K$ such that $L : \mathbb{Q}$ is radical.
(c) The Galois group of the polynomial $X^4 + X^2 + 1$ over $\mathbb{Q}$ is solvable.
(d) Each radical extension is normal.