Single Choice 4

1. Let a, b be algebraic over $\mathbb{Q}$, such that $[\mathbb{Q}(a) : \mathbb{Q}] = 3$ and $[\mathbb{Q}(b) : \mathbb{Q}] = 5$. Then the possible degrees of $\mathbb{Q}(a, b)$ over $\mathbb{Q}$ are

   (a) $[\mathbb{Q}(a, b) : \mathbb{Q}] = 3$
   (b) $[\mathbb{Q}(a, b) : \mathbb{Q}] = 5$
   (c) $[\mathbb{Q}(a, b) : \mathbb{Q}] = 15$
   (d) All of the above.

2. Let $L : \mathbb{Q}$ be a field extention and $a, b \in L\backslash\{0\}$ such that $a + b \neq 0$. Which of the following statements is **false**?

   (a) $a^2 \in \mathbb{Q}(a + b, ab) \Rightarrow a \in \mathbb{Q}(a + b, ab)$
   (b) $a \in \mathbb{Q}(a + b, ab) \Rightarrow a^2 \in \mathbb{Q}(a + b, ab)$
   (c) $[\mathbb{Q}(a, b) : \mathbb{Q}(a + b, ab)]$ is equal to the degree of the minimal polynomial of a over $\mathbb{Q}(a + b, ab)$.
   (d) All the statements above are true.

3. Let $M : L : K$ be field extentions and assume that a is algebraic over M, L and K. Then

   (a) $m_{a,M} \mid m_{a,L}$ in $M[x]$
   (b) $m_{a,L} \mid m_{a,K}$ in $L[x]$
   (c) $m_{a,M} \mid m_{a,K}$ in $M[x]$
   (d) All the statements above are true.

4. Consider a field extention $\mathbb{Q}(a, b) : \mathbb{Q}$. Which of the following statements is **false**?

   (a) If $\mathbb{Q}(a, b) : \mathbb{Q}$ is algebraic, then also a and b are algebraic over $\mathbb{Q}$.
   (b) If $\mathbb{Q}(a + b) : \mathbb{Q}$ and $\mathbb{Q}(ab) : \mathbb{Q}$ are algebraic, then also a and b are algebraic over $\mathbb{Q}$.
   (c) If a is transcendental over $\mathbb{Q}(b)$, then a is also transcendental over $\mathbb{Q}$.
   (d) If a is transcendental over $\mathbb{Q}$, then a is also transcendental over $\mathbb{Q}(b)$.

5. Let a, b be algebraic over $\mathbb{Q}$, such that the minimal polynomial of a and b both have degree 2. Then the degree of the minimal polynomial of $a + b$ is ...

   (a) 2
   (b) 4
   (c) a divisor of 2
   (d) a divisor of 4