

## Single Choice 6

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- Which of the fields below are a splitting field of the polynomial  $X^4 - 3$  over  $\mathbb{Q}$ ?
  - $\mathbb{Q}(\sqrt[4]{3}, i)$
  - $\mathbb{Q}(\sqrt[4]{3}, i\sqrt[4]{3})$
  - $\mathbb{Q}(\sqrt[4]{3}, i\sqrt{3})$
  - All of the above.
- Let  $K$  be a field. Which of the following statements is **false**?
  - If  $K$  has no proper algebraic extensions, then every non-constant polynomial  $f \in K[X]$  has at least one root in  $K$ .
  - If each irreducible polynomial  $f \in K[X]$  is linear, then  $K$  is algebraically closed.
  - If  $K_1$  and  $K_2$  are algebraic closures of  $K$ , then  $K_1$  and  $K_2$  are isomorphic over  $K$ .
  - If  $K$  contains a subfield which is algebraically closed, then  $K$  is algebraically closed as well.
- Which field extension is normal?
  - $\mathbb{F}_2(X) : \mathbb{F}_2(X^3)$
  - $\mathbb{F}_5(X) : \mathbb{F}_5(X^5)$
  - $\mathbb{Q}(\sqrt[4]{5}) : \mathbb{Q}$
  - $\mathbb{R} : \mathbb{Q}$
- The statement: *The field extension  $\mathbb{Q}(\sqrt{2 + \sqrt{2}}) : \mathbb{Q}$  is normal, is...*
  - true
  - false
- Over which field is the polynomial  $X^3 + 1$  separable?
  - $\mathbb{Q}$
  - $\mathbb{R}$
  - $\mathbb{F}_5$
  - All of the above.