Single Choice 8

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1. Which of the following statements is true for the Galois group $\mathrm{Gal}(\mathbb{Q}(\sqrt{3}, i) : \mathbb{Q})$?

(a) $\mathrm{Gal}(\mathbb{Q}(\sqrt{3}, i) : \mathbb{Q}) \cong \mathbb{Z}/4\mathbb{Z}$

(b) $\mathrm{Gal}(\mathbb{Q}(\sqrt{3}, i) : \mathbb{Q}) \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$

(c) $\mathrm{Gal}(\mathbb{Q}(\sqrt{3}, i) : \mathbb{Q}) \cong S_4$

(d) None of the above.

2. Let $H \leqslant \mathrm{Gal}(\mathbb{Q}(\sqrt{3}, i) : \mathbb{Q})$ be a subgroup of order 2, i.e. $|H| = 2$. Then the fixed field $\mathbb{Q}(\sqrt{3}, i)^H$ is only given by...

(a) $\mathbb{Q}(i)$.

(b) the fields $\mathbb{Q}(i)$ and $\mathbb{Q}(\sqrt{3})$.

(c) the fields $\mathbb{Q}(i), \mathbb{Q}(i\sqrt{3})$ and $\mathbb{Q}(\sqrt{3})$.

(d) the fields $\mathbb{Q}, \mathbb{Q}(i), \mathbb{Q}(i\sqrt{3})$ and $\mathbb{Q}(\sqrt{3})$.

3. Between which field extensions does there exist a field homomorphism over $\mathbb{Q}$?

(a) $\mathbb{Q}(i) \to \mathbb{Q}(\pi)$

(b) $\mathbb{Q}(\sqrt[3]{9}) \to \mathbb{Q}(\sqrt[3]{3})$

(c) $\mathbb{Q}(i) \to \mathbb{Q}(\sqrt{3})$

(d) $\mathbb{Q}(\sqrt{3}) \to \mathbb{Q}(\sqrt[3]{3})$

4. The order of the Galois group $\mathrm{Gal}(\mathbb{F}_{27} : \mathbb{F}_3)$ is

(a) 1

(b) 2

(c) 3

(d) 4

5. Which of the following statements are **false**?

(a) Let $L : M : K$ be field extensions. Then $\mathrm{Gal}(L : M) \leqslant \mathrm{Gal}(L : K)$.

(b) Let L be a splitting field of a polynomial over K. Then $|\mathrm{Gal}(L : K)| = [L : K]$.

(c) For $L : K$ a finite field extension, there exists $n \geqslant 1$ and an embedding $\mathrm{Aut}_K(L) \hookrightarrow S_n$.

(d) $\mathrm{Gal}(\mathbb{F}_{2^2} : \mathbb{F}_2)$ is generated by the homomorphism $\mathbb{F}_{2^2} \to \mathbb{F}_{2^2}, x \mapsto x^2$.