Single Choice 9

1. The order of the Galois group $|\operatorname{Gal}(\mathbb{Q}(\sqrt{7},\sqrt{11}):\mathbb{Q})|$ is equal to

(a) 2
(b) 4
(c) 6
(d) 8

2. Let $\alpha := \sqrt{7}+\sqrt{11}$. Let $\alpha_0,\dots,\alpha_r$ be the images of α under all the automorphisms of $\operatorname{Aut}(\mathbb{Q}(\sqrt{7},\sqrt{11}):\mathbb{Q})$. Then the product $\alpha_0\cdots\alpha_r$ is equal to

(a) -4
(b) $18+\sqrt{7\cdot 11}$
(c) $18-\sqrt{7\cdot 11}$
(d) 16

3. Which of the following fields is not normal over $\mathbb{Q}$?

(a) $\mathbb{Q}(\sqrt{11},\sqrt{13})$
(b) $\mathbb{Q}(e^{2\pi i/11})$
(c) $\mathbb{Q}(2^{1/11})$
(d) $\mathbb{Q}(\sqrt{11+\sqrt{13}},\sqrt{11-\sqrt{13}})$

4. The subgroup $H \leqslant \operatorname{Gal}(\mathbb{Q}(\sqrt{2},\sqrt{3},\sqrt{5}):\mathbb{Q})$ for which

$$\mathbb{Q}(\sqrt{2},\sqrt{3},\sqrt{5})^H = \mathbb{Q}(\sqrt{30})$$

is isomorphic to

(a) $\mathbb{Z}/2\mathbb{Z}$
(b) $\mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}$
(c) $\mathbb{Z}/4\mathbb{Z}$
(d) $\mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/4\mathbb{Z}$

5. Let $L:K$ be a finite Galois extension, such that $G=\operatorname{Gal}(L:K)\cong\mathbb{Z}/6\mathbb{Z}$. Let H be a subgroup of G of order 2, and Q be a subgroup of G of order 3. Which of the following statements is false?

(a) $[L^H:K]=\dfrac{[L:K]}{3}$
(b) $[L:K]=6$
(c) $L^H\neq L^Q$
(d) $L:K$ is separable.