Solutions Single Choice 2

1. In the ring $\mathbb{Z}[i]$, we have $\gcd(i, 1-i, 5)$ is given by

(a) 5
(b) $1-i$
(c) 2
(d) 1

Solution: We have the the units of $\mathbb{Z}[i]$ are given by $\{\pm 1, \pm i\}$, hence i is a unit in $\mathbb{Z}[i]$, so the greatest common divisor has to be a unit as well. Hence is (d) the correct answer.

2. Which of the following statements is wrong?

(a) Each principal ideal domain is a unique factorization domain.
(b) Each Euclidean ring is an integral domain.
(c) Each Euclidean ring is a principal ideal domain.
(d) Each unique factorization domain is a Euclidean ring.

Solution: The correct answer is (d): see notes of Lecture 2.

3. Which of the following rings is **not** a principal ideal domain?

(a) $\mathbb{R}[X]$
(b) $\mathbb{Z}[X]$
(c) $\mathbb{Z}[X]/(X^2+1)$
(d) $\mathbb{R}$

Solution: The correct answer is b). Note that $\mathbb{Z}[X]/(X^2+1) \cong \mathbb{Z}[i]$ and we have seen in the lecture that this is a PID. We have seen in the lecture as well that any field R is a PID, just like $R[X]$, for R a field.

4. Which of the following elements in $\mathbb{Z}[i]$ are irreducible?

(a) $2+i$
(b) $1+3i$
(c) $3+i$
(d) All of the above.

Solution: Only $2+i$ is irreducible. Note that $3+i = (1-i)(1+2i)$ and $1+3i = (1+i)(2+i)$.

5. Let K be a field and let $K[t^2, t^3]$ be the subring of $K[t]$ generated by t^2 and t^3. Which of the following statements is true?

(a) t^2 is irreducible in $K[t^2, t^3]$.

(b) t^2 is prime in $K[t^2, t^3]$.

(c) Every irreducible element is a prime element in $K[t^2, t^3]$.

(d) All of the above.

Solution: The correct answer is a). For b, note that t^2 divides $t^3 \cdot t^3$, but does not divide t^3 in $K[t^2, t^3]$. Thus not every irreducible element is prime in $K[t^2, t^3]$.