For each of the following two questions, select *all* correct answers. There is at least one correct answer, and possibly more than one. A fully correct answer gives **one point**, if there is one mistake, it gives $\frac{1}{2}$ *point*, and if there are two mistakes or more, it gives *zero point*.

(1) Which of the following properties of connected spaces is true?
   (a) If X is connected and $Y \subset X$, then Y is connected.
   (b) If X is a connected metric space, then it is locally connected.
   (c) If X is connected, then all elements of X have the same connected component.
   (d) If X is connected and $f\colon Y \to X$ is continuous and not constant, then $f^{-1}(X)$ is connected.

(2) For $n \geq 1$, let $X_n = [-1, 1]$ with the euclidean topology and let

$$X = \prod_{n \geq 1} X_n$$

with the product topology. Which of the following statements are true?
   (a) The space X is not locally compact.
   (b) The space X is connected.
   (c) The map $f\colon X \to X$ defined by

$$f(x_1, x_2, \ldots, x_n, \ldots) = (x_2, x_4, \ldots, x_{2^n}, \ldots)$$

   is continuous.
   (d) The map $g\colon \mathbf{R} \to X$ defined by

$$g(t) = (\cos(nt))_{n \geq 1}$$

   is continuous.