

For each of the following two questions, select *all* correct answers. There is at least one correct answer, and possibly more than one. A fully correct answer gives **one point**, if there is one mistake, it gives  $\frac{1}{2}$  *point*, and if there are two mistakes or more, it gives *zero point*.

- (1) Let  $X$  be a normal topological space. Which of the following properties are true?
- (a) Every  $x \in X$  has a fundamental system of closed neighborhoods.
  - (b) If  $a \neq b$  are points of  $X$ , then there is a continuous function  $f: X \rightarrow \mathbf{C}$  such that  $f(x) = 1$  for all  $x$  in a neighborhood of  $a$  and  $f(b) = i$ .
  - (c) For all closed subsets  $A$  and  $B$  in  $X$ , there exists a continuous function  $f: X \rightarrow [-1, 1]$  such that  $f(x) = 1$  for all  $x \in A$  and  $f(x) = -1$  for all  $x \in B$ .
  - (d) For all closed subset  $A$  and open set  $U$  in  $X$  with  $U \cap A = \emptyset$ , there exists a continuous function  $f: X \rightarrow [-1, 1]$  such that  $f(x) = 1$  for all  $x \in A$  and  $f(x) = -1$  for all  $x \in U$ .
- (2) Let  $X = [-1, 1] \times [-1, 1]$  with the product topology, and let  $\sim$  be the equivalence relation where

$$(a, b) \sim (c, d) \text{ if and only if } a^2 + b^2 = c^2 + d^2.$$

Let  $Y = X/\sim$  with the quotient topology and let  $p: X \rightarrow Y$  be the projection. Which of the following properties are true?

- (a)  $Y$  is compact.
- (b)  $Y$  has four connected components.
- (c) The subspace
 
$$p(\{(x, y) \in X \mid -1/2 \leq x \leq 1/2, \quad x^2 + y^2 > 1\}) \subset Y$$
 is connected.
- (d)  $Y$  is not locally connected.