

For each of the following two questions, select *all* correct answers. There is at least one correct answer, and possibly more than one. A fully correct answer gives **one point**, if there is one mistake, it gives  $\frac{1}{2}$  *point*, and if there are two mistakes or more, it gives *zero point*.

(1) Which statements are correct?

- (a) The map  $f: \mathbf{R}^\times \rightarrow ]0, +\infty[$  given by  $f(x) = x^2$  is a trivializable covering space.
- (b) The map  $f: \mathbf{R} \rightarrow [0, +\infty[$  given by  $f(x) = x^2$  is a universal covering space.
- (c) The map  $f: \mathbf{R}^2 \rightarrow \mathbf{S}_1 \times \mathbf{S}_1$  given by  $f(s, t) = (e^{2i\pi t}, e^{2i\pi s})$  is a universal covering space.
- (d) The map  $f: \mathbf{C}^\times \rightarrow \mathbf{C}^\times$  given by  $f(z) = 1/z^3$  is a trivializable covering space.

(2) Which of the following are correct?

(a) The space

$$\{z \in \mathbf{C} \mid 1 < |z| < 2\} \subset \mathbf{C}$$

is simply connected.

(b) The space

$$\{(x_1, \dots, x_n) \in \mathbf{R}^n \mid x_n > 0\} \subset \mathbf{R}^n,$$

where  $n \geq 1$  is an integer, is simply-connected.

(c) The quotient space  $\mathbf{S}_2/G$ , where

$$\mathbf{S}_2 = \{(x, y, z) \in \mathbf{R}^3 \mid x^2 + y^2 + z^2 = 1\}$$

and the group  $G = \{-1, 1\}$  acts by  $\varepsilon \cdot (x, y, z) = (\varepsilon x, \varepsilon y, \varepsilon z)$ , has fundamental group at any point isomorphic to  $\mathbf{Z}/2\mathbf{Z}$ .

(d) The quotient space  $\mathbf{R}^2/\mathbf{Z}$ ,  $\mathbf{Z}$  acts by  $n \cdot (x, y) = (x + n, y + 2n)$ , has fundamental group at any point isomorphic to  $\mathbf{Z}^2$ .