

The oral exam will start with one randomly chosen question from this list, and continue with questions that might not be on the list.

- (1) How is the tensor product of modules defined?
- (2) How do you compute tensor products of finitely generated abelian groups?
- (3) Which short exact sequences remain exact when tensored with a module?
- (4) How does one construct homology with coefficients?
- (5) What does it mean that the connecting homomorphism for homology with coefficients is natural?
- (6) Show that $\mathbb{R}P^m$ is not a retract of $\mathbb{R}P^n$ for $1 < m < n$.
- (7) State the theorem of Borsuk–Ulam. What is the idea of the proof?
- (8) How is Ext defined?
- (9) How does one prove that Tor is commutative? Why does this proof not work for Ext?
- (10) Does the Euler characteristic depend on the coefficient field?
- (11) Compute $H_\bullet(\mathbb{R}P^n; \mathbb{F}_2)$ from $H_\bullet(\mathbb{R}P^n; \mathbb{Z})$ using the universal coefficient theorem for homology.
- (12) Prove that $H^1(X)$ is torsion-free for all spaces X .
- (13) How is the relative cup product defined?
- (14) Compute $H^\bullet(S^k)$.
- (15) What does it mean that “the cup product is non-singular”?
- (16) Give an example for spaces with isomorphic cohomology groups, but non-isomorphic cohomology ring.
- (17) How does one compute the cohomology ring of the torus?
- (18) What can you say about the fundamental group of an orientable manifold?
- (19) What is the fundamental class?
- (20) Is there an orientation reversing homeomorphism of $\mathbb{C}P^2$?
- (21) Can two manifolds of different dimension be homotopy equivalent?
- (22) Is there a relationship between \smile and \frown ?
- (23) State Poincaré duality, also for non-compact manifolds.
- (24) What can you say about the Euler characteristic of a 5-manifold?
- (25) Compute the cohomology ring $H^\bullet(\mathbb{R}P^n; \mathbb{F}_2)$.
- (26) Give an example of a direct limit.
- (27) How is cohomology with compact support H_c^k defined?
- (28) How is H_c^k covariantly functorial?
- (29) What does Alexander duality say? What is the idea of the proof?
- (30) Does there exist an embedding of the Klein bottle into \mathbb{R}^3 ?